Modeling and Control of Stand-alone and Parallel Operating Diesel Generators

by

Mohammad Abdulqader

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

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DEANSHIP OF GRADUATE STUDIES

This thesis, written by

MOHAMMAD ABDULQADER

under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of

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Thesis Committee

Dr. Maamar Bettayeb (Chairman)

Dr. A.H. Abdurahim (Co-advisor)

Dr. Yousef Abdelmagid (Member)

Dr. Ibrahim El-Amin (Member)

Dr. Samir Al-Baiyat
Department Chairman

Dr. Abdullah M. Al-Shehri
Dean of Graduate Studies

Date 21/5/2002

عمادة الدراسات العليا
DEANSHIP OF GRADUATE STUDIES

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****** Dedicated to my Family ******

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Abstract

Name : **Mohammad Abdulqader**

Title : **Modeling and Control of Stand-alone and Parallel Operating Diesel Generators**

Major Field : **Electrical Engineering**

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The stand-alone diesel generator supplying a local load model was developed along with its governor and excitation control systems, the model was tested with step load change as well as with step speed and voltage reference settings. The test results is compared with an actual one obtained from the real experiment. Then the model of the two diesel generators operating in parallel was also developed and tested with step load change as well as with step speed and voltage reference settings.

The load sharing principle was presented and its importance was emphasized. The load sharing control was developed and incorporated in the two parallel generators model. Then it was tested.

LQR controllers were designed for the stand-alone and parallel diesel generators state feedback. The test showed that LQR controllers improve the response of the models. Centralized LQR controller was also designed for parallel generators case, which showed good improvement in system performance. De-Centralized controller was also designed and tested. The test showed it is not suitable for this system. LQR controller with singular values-based weights was implemented, which uses systematically calculated Q and R matrices. This controller was tested on the parallel generators model. The computer simulation showed further improvement in eliminating transients.

King Fahd University of Petroleum & Minerals

Dhahran, Saudi Arabia

خلاصة الرسالة

الاسم : **محمد رفيق محمد عبد القادر**

عنوان الرسالة : **صياغة الأنماط الرياضية والتحكمية لمولدات الديزل التي تعمل على انفراد أو على التوازي**

التخصص : **هندسة كهربائية**

التاريخ : **صفر ١٤٢١ الموافق مايو ٢٠٠٠**

تم في هذا البحث تطوير نمط تحكمي لمولد ديزل منفرد يغذي حمل محلي بحيث اشتمل النمط على أنظمة التحكم بالسرعة والجهد. لقد تم اختبار محاكاة لاستقرارية النمط التحكمي وذلك بتعريضه لاضطراب ناتج عن زيادة مفاجئة لقيمة الحمل وبالمثل أيضا لقيم السرعة و الجهد المرجعيتين. تمت مقارنة نتائج الاختبار مع نتائج اختبار حقيقي تم إجراؤه لمولد مماثل. من ثم تم تطوير نمط تحكمي لمولدي ديزل يعملان على التوازي وتم اختبار النمط التحكمي بطريقة مماثلة لحالة المولد المنفرد.

تم استعراض مبدأ اقتسام الأحمال بين المولدات العاملة على التوازي والتشديد على أهميته حيث تم تطوير أنماط تحكمية لضبط اقتسام الأحمال الفعالة والأحمال الغير فعالة لكل مولد من المولدات العاملة على التوازي وبعد ذلك تم إدماج هذه الأنماط ضمن النمط العام للمولدات ومن ثم اختبارها.

تم في هذا البحث استخدام المنظمات الخطية من الدرجة الثانية (**LQR**) حيث صمم منظم لكل من المولد المنفرد ومنظم آخر مركزي للمولدين العاملين على التوازي باستخدام طرق التصميم التقليدية لمنظمات **LQR** وتم اختبار أنماط المولدات المنفردة والمتوازية بها والتي أبدت تحسن ملموس جدا لأداء أنماط المولدات عند تعريضها لاضطراب في الأحمال. من ثم تم تطوير **LQR** لامركزي للحالة المتوازية حيث بينت نتائج اختبار المحاكاة عدم قدرة هذا الأخير على إبقاء النظام في حالة مستقرة. بعد ذلك تم تصميم منظم **LQR** غير تقليدي يعتمد أساسا على حساب المصفوفتين **Q** و **R** بالاعتماد على القيم النسبية الموزونة للقيم المفردة (**Singular values-based weights**) ومن ثم تم اختبار المنظم حيث بينت النتائج زيادة مضطردة في تحسن الأداء العام.

درجة الماجستير في العلوم

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الظهران , المملكة العربية السعودية

صفر ١٤٢١ / مايو ٢٠٠٠

List of Symbols

X_d Generator direct axis synchronous reactance, p.u

X_d' Generator direct axis transient reactance, p.u

X_q Generator quadratic axis synchronous reactance, p.u

τ_{do}' Generator Open Circuit Field transient time constants, seconds

X_L Load reactance, p.u

R_L Load resistance, p.u

Z_L Load impedance, p.u

I_d Generator load direct axis current component, p.u

I Generator load current, p.u

I_q Generator load quadratic axis current component, p.u

V_d Generator terminal voltage d-axis component, p.u

V_q Generator terminal voltage q-axis component, p.u

E Machine internal voltage

E_q Q-axis internal voltage

E_q' Q-axis transient internal voltage

V_t Generator terminal voltage, p.u

P_e Generator output active power, p.u

Q_e Generator output reactive power, p.u

M Generator rotor inertia constant, MJ/MVA or seconds

WK^2 Moment of inertia, $lb.ft^2$ or $Kg.m^2$

n Rotor nominal speed, rpm

S_{mach} Synchronous machine rated apparent power, MVA

P_D Damping power, p.u

D Damping constant,

ω Rotor speed, p.u

δ Rotor angle, radian/sec or degrees

E_{fd} Generator field voltage, p.u

K_l Generator(s) load constant

K_V Generator terminal voltage constant, p.u

K_P Generator active power constant, p.u

K_Q Generator reactive power constant, p.u

V_e Terminal voltage error, p.u

V_S Stabilizer voltage, p.u

V_F Excitation Feedback voltage, p.u

V_R Regulator voltage, p.u

K_A Voltage regulator amplifier gain

T_A Voltage regulator amplifier time constant, seconds

T_E Exciter time constant, seconds

K_F Feedback circuit gain

T_F Feedback circuit time constant, seconds

K_E Exciter gain

V_{REF} Voltage reference setting, p.u

S_E Rotating exciter saturation factor, p.u

X_A Governor actuator state

X_G Governor controller state

P_m	Rotor mechanical power, p.u
K_1	Engine torque constant, p.u
K_2	Governor actuator constant, p.u
K_3	Actuator current driver constant, p.u
τ_1	Diesel engine dead time constant, seconds
τ_2	Governor actuator time constant, seconds
S	Laplace operator = $j\omega$
K_G	Speed controller proportional gain
K_D	Speed controller derivative gain
K_{IN}	Speed controller integral gain
ω_{REF}	Speed reference setting, p.u
P_L	Load active power, p.u
Q_L	Load reactive power, p.u
f	Generator frequency, p.u
Δ	Incremental change
K_q	Field voltage constant
X_1-X_{50}	Generator loading reactance constants, p.u
X_{1i}-X_{50i}	Generator initial loading reactance constants, p.u

Chapter 1

INTRODUCTION

1.1 Diesel Generators in Power Systems:

Diesel generators have been used as a reliable source of electric power for emergency situations and also as prime power generating plants around the world. In Saudi Arabia more than 700 MW of prime power is generated though diesel power plants and sold through SCECO companies, besides a large number of private power generating plants ranging from 4 MW up to 50 MW capacity each.

The particularity and specialty of Diesel Engine-driven generator sets as distinguished from other conventional generators can be summarized [1,2,3] as follows:

1. Used mainly in emergency power applications or where sudden demands for back-up power are expected.
2. Low inertia fast responding prime movers.
3. Highly sensitive to load changes and faults which has a direct impact on the system frequency for multi-unit generation.
4. It has largely non-linear time-varying operating characteristics.

5. Presence of an input dead time (unknown time delay) between the actuator oil injection and the mechanical torque production, and this dead time is a non-linear function of operating conditions and engine speed.
6. The noise component of the shaft torque is much greater than other prime movers.
7. Large diesel prime movers dynamic response is significantly governed by the dynamics and inertia of the turbocharger, which might cause oscillatory dynamic response.

When operating a number of diesel generators in parallel, some problems arise, most of which are well known to the power system engineers and operators from the operation side but lacks in understanding from performance and control sides. These problems are mainly how to maintain frequency and terminal voltage of each generator at rated values after transient disturbances. Also when number of diesel generators are in parallel the active and reactive load must be shared between them.

1.2 Literature Review:

The co-ordination of speed, voltage control and load sharing aspects of multiple diesel generating units have not been well covered in the literature. Many research work concentrated on developing speed controls of the diesel generators [1,2,3,4].

Another research discussed developing output feedback controller for the frequency and voltage of a stand-alone diesel generator [5]. This did not cover the parallel generators case and what happens to the electrical powers. Few papers discussed the diesel generator set modeling and control problem. Non-linear full model was developed for stand-alone diesel generator set using non-linear diesel engine model and type-I excitation system [6,7]. The model was reduced to the most dominant states, the simulation results showed that the reduced model has fairly accurate representation. In [6,7] the authors did not address the active and reactive power output issues of the diesel generator set. In [8] the authors developed the dynamic equations for two machines, one of them diesel geneator and the other one is hydro generator connected to each other through a short transmission line. The models and controls developed in [6,7] were used here. The P and Q and their sharing in parallel was not addressed, and also the case of having more than one diesel generators on the same bus was not addressed as well. In [9] diesel generator models were developed along with asynchronous wind-turbine models for multi-machine case to connect them to the utility system. Again P and Q control was not addressed. In [10] the authors developed equivalent machine model for multi-diesel generator system for the purpose of designing an optimum controller. The issue of active and reactive power load sharing along with speed and voltage control was addressed but it did not show the validity of the proposed algorithm if the number of states changes or if the generators were different. The governor and excitation controls were not included. The application used Silicon Controlled Rectifier (SCR) loads for Oil Rigs and did not investigate the case with normal load disturbances.

1.3 Research Objective:

The problem of diesel generators becomes obvious by knowing that lot of consultant offices and customers are increasingly asking for minimum requirements in transient performance of these generators, which could not be achieved usually without seeking the help from the equipment original manufacturers. As a typical example of such requirements the diesel generator must be capable of accepting a sudden step load of 50%-100% while its speed recovery time should not exceed 5-10 seconds. In some cases customers and/or consultants ask for transient performance curves and certificates from the generator manufacturer, which of course can not be given simply because the performance of most of these generators vary from one generator to another depending on the diesel engine, alternator, governor and excitation system which are used. The only practical way is to measure this performance on the test line using chart recorders and calibrated instruments every time this is required.

Every diesel manufacturer uses different combinations of equipment to build a diesel generator for a customer. The diesel engine manufacturer is totally different from the ac alternator manufacturer. The governor manufacturer is also different from the engine manufacturer and excitation system manufacturer. All this variety of combination of different equipment creates special cases every time a generator is built for a certain customer.

This is a trial to study the diesel generators modeling and controls issues as one topic with trials to implement centralized controllers to improve their dynamic transients. One of the main objectives is to spot light on the areas, which were either not covered or poorly covered in the literature. These can be summarized as:

1. Step by step modeling of the complete diesel generators using brushless excitation models and PID speed control models, being the two commonly used excitation and speed controls for the nowadays diesel generators.
2. The stand-alone and parallel generators connected to local load cases will be considered.
3. Active and reactive powers dynamic responses.
4. Response to application of resistive and reactive loads on diesel generators
5. Isochrounous speed governing characteristics
6. Active and reactive load sharing

1.4 Thesis Outlines:

This thesis is just a trial to simplify the diesel generator modeling and control problem for the industrial engineers working in this field and give basic reference material for graduate students who wish to do further studuies on the dynamic performance of diesel generators. The simulation in this thesis will emphasize on the cases that have great importance to industrial control system design engineers, testing engineers and consulting engineers. Special emphasis is given on parallel operation and on the active and reactive power load-sharing problem.

In Chapter-2 the stand-alone diesel generator is modeled using simple but practical model followed by test of the non-linear model and for the linear model. A practical experiment is performed on a diesel generator, which has similar parameters and rating as the study generators. This experiment is done in Saudi Diesel Generators Co. premises in Al-Khobar, Saudi Arabia. The response of this generator is recorded when it is applied with sudden load. The results are compared with the model simulation. In Chapter-3 the case of parallel diesel generators is modeled by taking example of two generators. The models are then tested the same way as in Chapter 2. In Chapter-4 the principle of active and reactive power load sharing is presented and modeled with some emphasis on the difference between isochronous and droop control. In Chapter-5 the stand-alone and parallel generator models are tested with LQR controls. Then a LQR algorithm is implemented using calculated Q and R mastrices with system's singular values-based weights [17]. Chapter-6 concludes the thesis work and results and give recommendations for future work. Matlab programs written for the tests of the models of Chapters 2,3, 4 and 5 are listed in Appendix D.

Chapter 2

MODELING OF STAND-ALONE DIESEL GENERATOR

The dynamic model of a simple diesel generator connected to its local load is developed in this chapter. The generator with all its control elements are shown in Fig 2.1

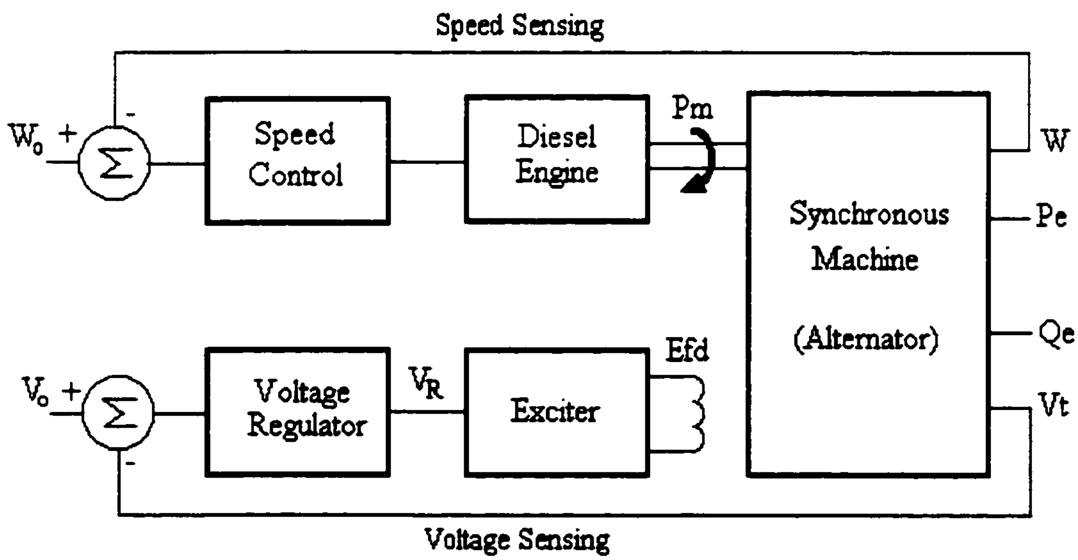

Fig 2.1 Diesel generator functional block diagram

The synchronous generator has two inputs, the mechnical power P_m supplied by the diesel engine having its own speed control mechanism, and the field excitation voltage E_{fd} supplied from the voltage regulator. The generator is connected to its local load as shown in Fig 2.2

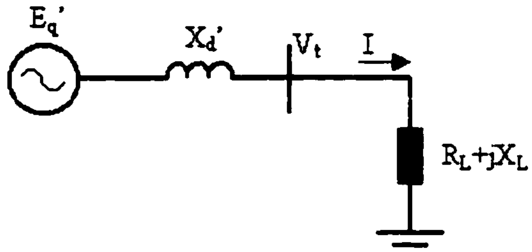

Fig 2.2 One generator connected to local load

The composite system equations derived in this chapter includes the nonlinear synchronous machine dynamics, the diesel engine model, the speed governor actuator model, the speed governor controller model, and the generator excitation and AVR models. The dynamics of each component is given in the following.

2.1 Synchronous Generator Model:

The synchronous generator is modeled through 3rd order dynamics. The states are rotor angle (δ), generator speed (ω) and the internal voltage of the generator (E_q'). The electromechanical swing equation is written as

$$\dot{\omega} = (1/M)[P_m - P_e] \qquad \text{p.u.} \tag{2.1}$$

$$\dot{\delta} = \omega_o[\omega - 1] \qquad \text{p.u} \tag{2.2}$$

where δ is the angle of the generator internal voltage E_q' with respect to the terminal voltage of the generator. P_m is the mechanical input power in p.u, P_e is the active electrical power in p.u, , M is the inertia constant in seconds or MJ/MVA, which is given by the manufacturer, who used to supply this information as moment of inertia WK^2 in $lb.ft^2$ or $kg.m^2$. M can be calculated for a certain machine as follows:

$$M = 4.62 \times 10^{-10}\, WK^2\, n^2/S_{mach} \quad MJ/MVA \tag{2.3}$$

Where WK^2 is the moment of inertia in $lb.ft^2$, see Table A.1 in Appendix-A for typical values for different diesel generator rotors. 'n' is the rated speed in r.p.m, and S_{mach} is the machine base MVA.

The system of equation (2.1) is represented in block diagram as in Fig 2.3

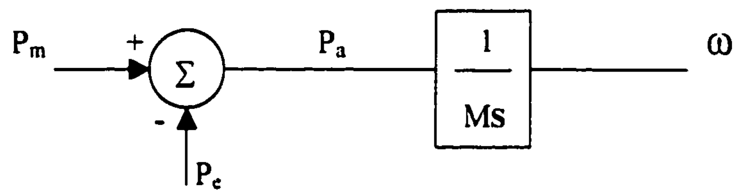

Fig 2.3 Swing Equation Representation

The expressions for the electrical power output P_e is

$$P_e = V_d I_d + V_q I_q \tag{2.4}$$

$$P_e = K_P E_q'^2 \tag{2.5}$$

Where K_P is

$$K_P = K_I^2 R_L x_q (x_L+x_q)+K_I R_L-K_I^2 R_L x_d'(x_L+x_q) \tag{2.6}$$

Where K_I is the reactance constant defined as

$$K_I = 1/[R_L^2+x_L(x_L+x_q)+x_d'(x_L+x_q)] \tag{2.7}$$

Similarly we write the reactive electrical power Q_e as

$$Q_e = V_q I_d - V_d I_q \tag{2.8}$$

$$= K_Q E_q'^2 \tag{2.9}$$

where K_Q is

$$K_Q = K_I (x_L+x_q) - K_I^2 x_d' (x_L+x_q)^2 - K_I^2 R_L^2 x_q \tag{2.10}$$

The voltage-current phasors diagram is shown in Fig 2.4

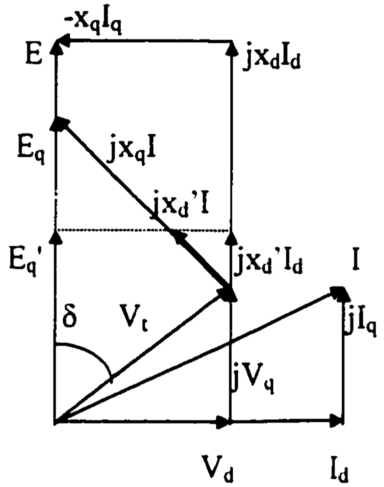

Fig 2.4 Third Order Generator Phasor Diagram

The internal voltage of the generator E_q' is expressed through the differential equation

$$\dot{E}_q' = (1/\tau_{do}')[E_{fd} - E_q' - (x_d - x_d')I_d] \tag{2.11}$$

Where E_{fd} is the field voltage in p.u., x_d is the direct axis synchronous reactance in p.u., x_d' is the direct axis transient reactance in p.u., I_d is the direct axis component of the load current I in p.u., and τ_{do}' is the open circuit transient field time constant in seconds.

I_d is written in terms of E_q' as

$$I_d = K_I (x_L+x_q) E_q' \tag{2.12}$$

2.2 Excitation System and AVR:

Considering diesel generators with brushless excitations systems. We shall consider IEEE Type-II (AC5A) excitation as shown in Fig 2.5.

This model has 3 states namely V_R, V_F and E_{fd} where V_R is the regulator state, V_F for the feedback stabilizing signal and E_{fd} the field voltage, which is the exciter output state. T_R is set to zero. The three state equations for the model of Fig 2.5 are

$$\dot{V}_R = -K_A V_t/T_A - V_R/T_A - (K_A V_F)/T_A + (K_A V_{REF})/T_A \tag{2.13}$$

$$\dot{E}_{fd} = V_R/T_E - (K_E+S_E)E_{fd}/T_E \tag{2.14}$$

$$\dot{V}_F = -K_A K_F V_t/(T_A T_F) - (K_A K_F+T_A)V_F/(T_A T_F) + K_A K_F V_{REF}/(T_A T_F) - K_F V_R/(T_A T_F) \tag{2.15}$$

Where T_A is the voltage regulator amplifier time constant, T_E is the exciter time constant, and S_E is the saturation constant.

The terminal voltage V_t in equations (2.12) and (2.14) is obtained in terms of E_q' as

$$V_t = \sqrt{V_d^2 + V_q^2}$$

$$= K_V E_q' \tag{2.16}$$

where K_V , V_d and V_q are

$$K_V = \sqrt{K_I^2 R_L^2 x_q^2 + [1-K_I x_d' (x_L+x_q)]^2} \tag{2.17}$$

$$V_d = K_I R_L x_q E_q' \tag{2.18}$$

$$V_q = [1 - K_I x_d'(x_L+x_q)]E_q' \tag{2.19}$$

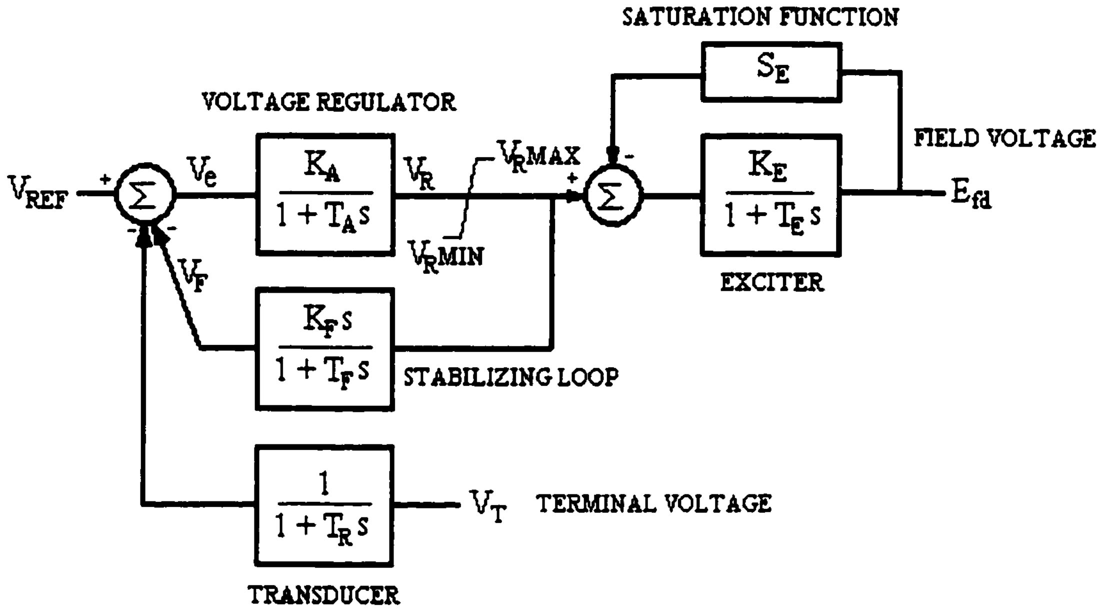

Fig 2.5 Excitation Control Model. IEEE Type II (AC5A)

2.3 Diesel Engine & Speed Control:

2.3.1 Diesel Engine:

Diesel engine is represented by a time delay with time delay constant τ_1 , which is referred to as the input dead time of prime mover and a gain K_1. These two parameters have non-linear nature [1] as a function of speed. It should be pointed out that larger order simulation models for the diesel engines have been reported in the literature [6,7,8]. However, for most studies on speed dynamics of internal combustion engines, a model similar to Fig 2.6 is considered to be adequate. Because transients are relatively fast, oscillatory modes concerning variables like temperature and pressure can usually be ignored [4]. As shown in Fig 2.6 the input state of the engine model is X_A while the output state is P_m.

Fig 2.6 gives the block diagram representation of a diesel engine and actuator system. This model was successfully used in the literature [3,4,5,7,9].

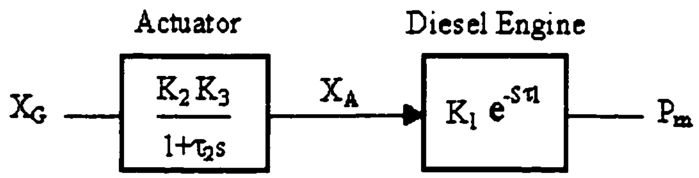

Fig 2.6 Diesel Engine Model Representation

The diesel engine is modeled by a simple time delay as

$$P_m/X_A = K_1\, e^{-S\tau 1} \qquad (2.20)$$

Which can be approximated by

$$\cong -K_1(s-2/\tau_1)/(s+2/\tau_1) \qquad (2.21)$$

The engine fuel actuator consists of two parts. The first part is the current driver, which converts the control signal produced by the controller to an equivalent current signal that drives the actuator. This current driver has a gain constant K_3. The actuator itself is an electrohydraulic device represented by a first order model with K_2 gain constant and time delay τ_2 that is dependent on the oil temperature. The output of the actuator is the fuel flow. The input state of the actuator is X_G and the output state is X_A as shown in Fig 2.6

is used in this study. The state equations for the diesel engine and actuator can then be written as,

$$X_A / X_G = K_2K_3/(1+s\tau_2) \tag{2.22}$$

From which we write the actuator state equation as,

$$\dot{X}_A = -(1/\tau_2)X_A + (K_2K_3/\tau_2)X_G \tag{2.23}$$

$$\dot{P}_m = -(2/\tau_1)P_m - K_1\dot{X}_A + (2K_1/\tau_1)X_A \tag{2.24}$$

From (2.22) and (2.23) we write the diesel engine state equation as,

$$\dot{P}_m = -(2/\tau_1)P_m + [K_1/\tau_2 + 2K_1/\tau_1]\, X_A - (K_1K_2K_3/\tau_2)\, X_G \tag{2.25}$$

2.3.2 Speed Controller:

A PID speed controller is used in most of the present day diesel generators. The block diagram representation is shown in Fig 2.7. The input-output equation of the controller is

$$X_G = (\omega_{REF}-\omega)\, [K_G + K_{IN}/S + K_DS] \tag{2.26}$$

Differentiating (2.26) and substituting for swing equation we get expression for $\dot{X}_G$ as,

$$\dot{X}_G = m_1 E_q'^2 + m_2E_q'E_{fd} + m_3\omega + m_4P_m - m_5X_A + m_6X_G + K_{IN}\omega_{REF} \tag{2.27}$$

Where m_1 to m_6 are constants, function of the machine's governor and excitation control constants. These are defined in Appendix-B.2.

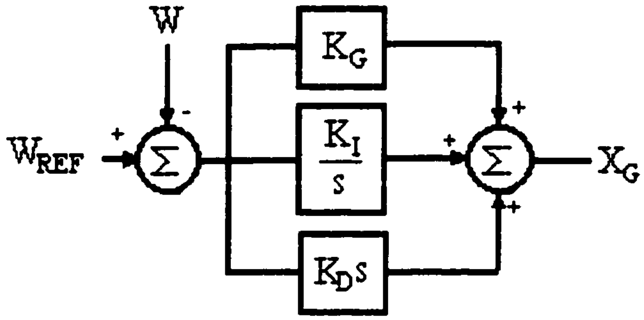

Fig 2.7 PID Type Speed Control

2.4 Complete Diesel Generator Model:

The complete model of the diesel generator can now be represented by 9 differential equations (2.1), (2.2), (2,11), (2.13), (2.14), (2.15), (2.23), (2.25) and (2.27) and written in the form

$$\dot{X} = f[x,u] \tag{2.28}$$

Where X is the vector of states $[\delta,\omega,E_q',E_{fd},V_R,V_F,X_A,X_G,P_m]^T$, and the control vector u comprises of $[\omega_{REF},V_{REF}]^T$.

2.5 Simulation Study:

The stand-alone diesel generator was simulated on Matlab. The set of nonlinear differential equations were solved using ODE4 toolbox . The algorithm is detailed in Appendix-D.1, and the system data is detailed in Appendix-A. The initial values of the states and other auxiliary equations are given in Appendix-B.1

2.5.1 Step Load Test:

The step change in the load is achieved by converting the load power change to resistance R_L and reactance X_L changes as detailed in Appendix B.1. We shall consider here 3 cases. In the first and second cases we change the power factor by changing either P_L or Q_L. The third case we change the load and maintain the power factor, which requires change of both P_L and Q_L.

CASE - 1: Step increase of 0.4 p.u in active load power P_L

The generator is initially operating at 0.4 p.u active load power. A step increase of P_L from 0.4 to 0.8 p.u is applied without changing the reactive load Q_L. These ratings are referred to generator # 1 base as detailed in Appendix A. The generator frequency, terminal voltage, active electrical power, and reactive electrical power responses are plotted and shown in Fig 2.8. The response shows 4 Hz dip in the frequency and receovered in less than 2 seconds. The terminal voltage increased sharply by 4% then it recovered in less than a second after one or two oscillations. The active power increased 0.4 p.u exactly as expected, which is the same amount of the load change while the reactive power maintained its value of 0.3 p.u although it was disturbed for a while as shown in Fig 2.8.

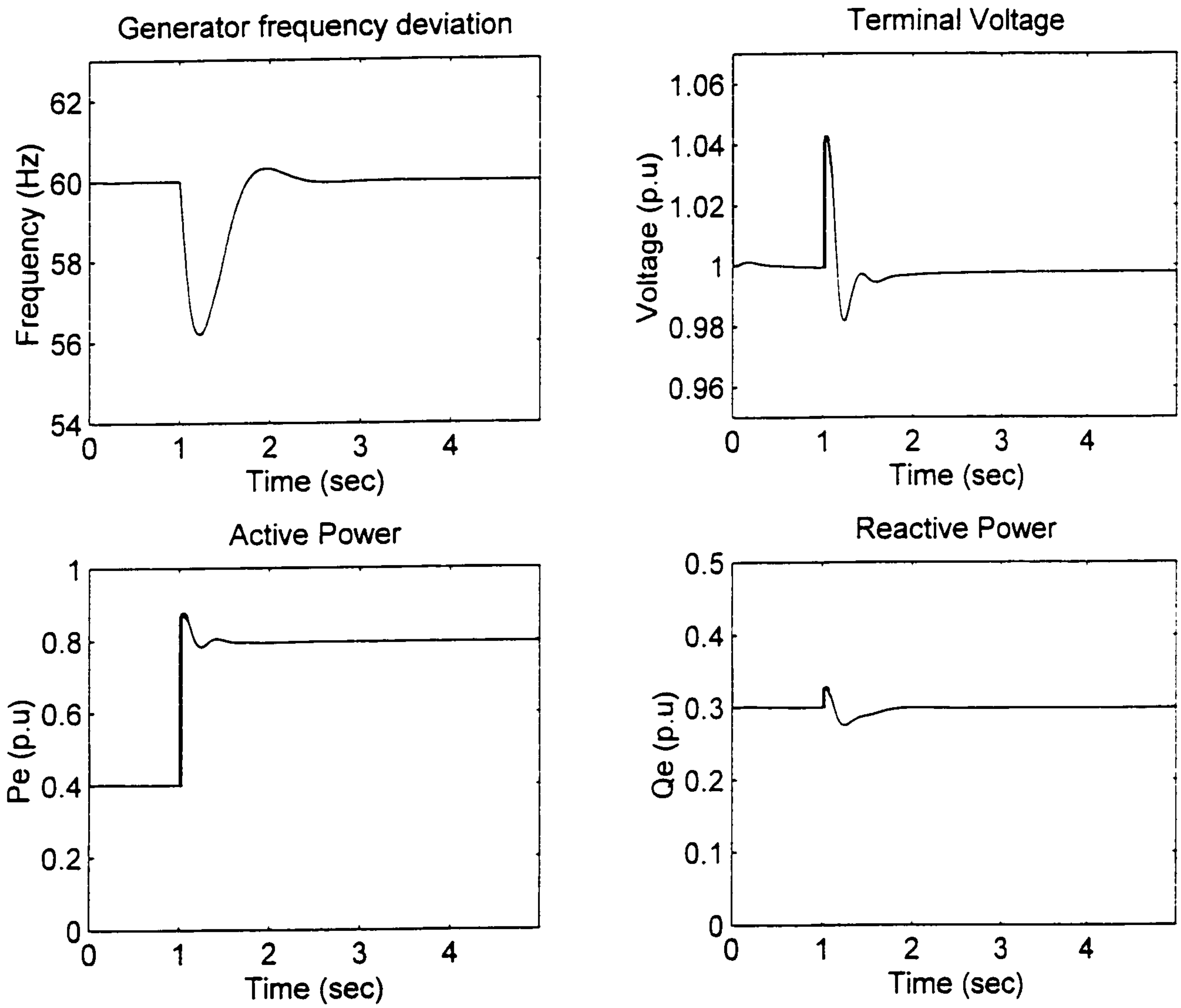

Fig 2.8 Response to step increase of 0.4 pu of active load power P_L

CASE - 2 : Step increase of 0.3 p.u in reactive load power Q_L

The generator is initially operating at 0.3 p.u reactive load power. A step increase in Q_L from 0.3 to 0.6 p.u is applied without changing the active load power P_L. The generator frequency, terminal voltage, active and reactive power responses are plotted and shown in Fig 2.9. The rsponse shows 6% dip in the voltage. By comparing this with case –1 (pure resistive load increase) we notice that purely reactive loads tends to decrease the voltage while resistive loads increase it. In this case the frequency deviation was very small (0.2 Hz). The reactive power increased by 0.3 p.u exactly the same value of the load change while the active power maintained its level of 0.4 p.u after 0.05 p.u dip and few oscillations.

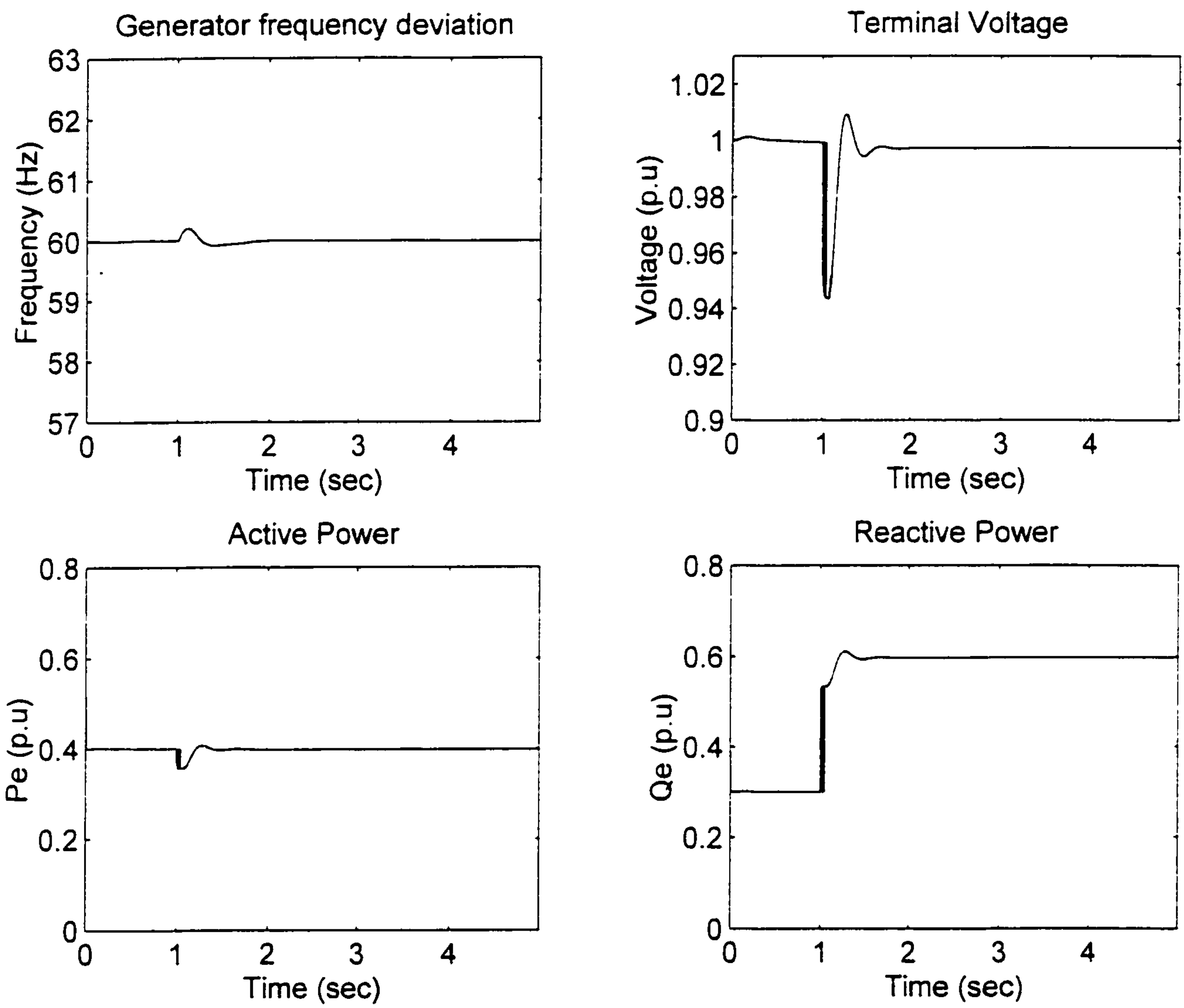

Fig 2.9 Response to step increase of 0.3 pu of Reactive load power Q_L

CASE - 3 : Step increase in active and reactive load P_L+jQ_L (constant power factor)

A step increase of the load P_L+jQ_L from 0.4+j0.3 to 0.8+j0.6 p.u. (no change in power factor) is applied. The frequency, terminal voltage, active and reactive electrical power responses are plotted and shown in Fig 2.10. Being a response of a combined resistive and reactive load we expect to see combination of the first two cases. Fig 2.10 shows that the frequency and the voltage both dip, 4 Hz for frequency and 3.5% for the voltage. The powers increase exactly to the new load level.

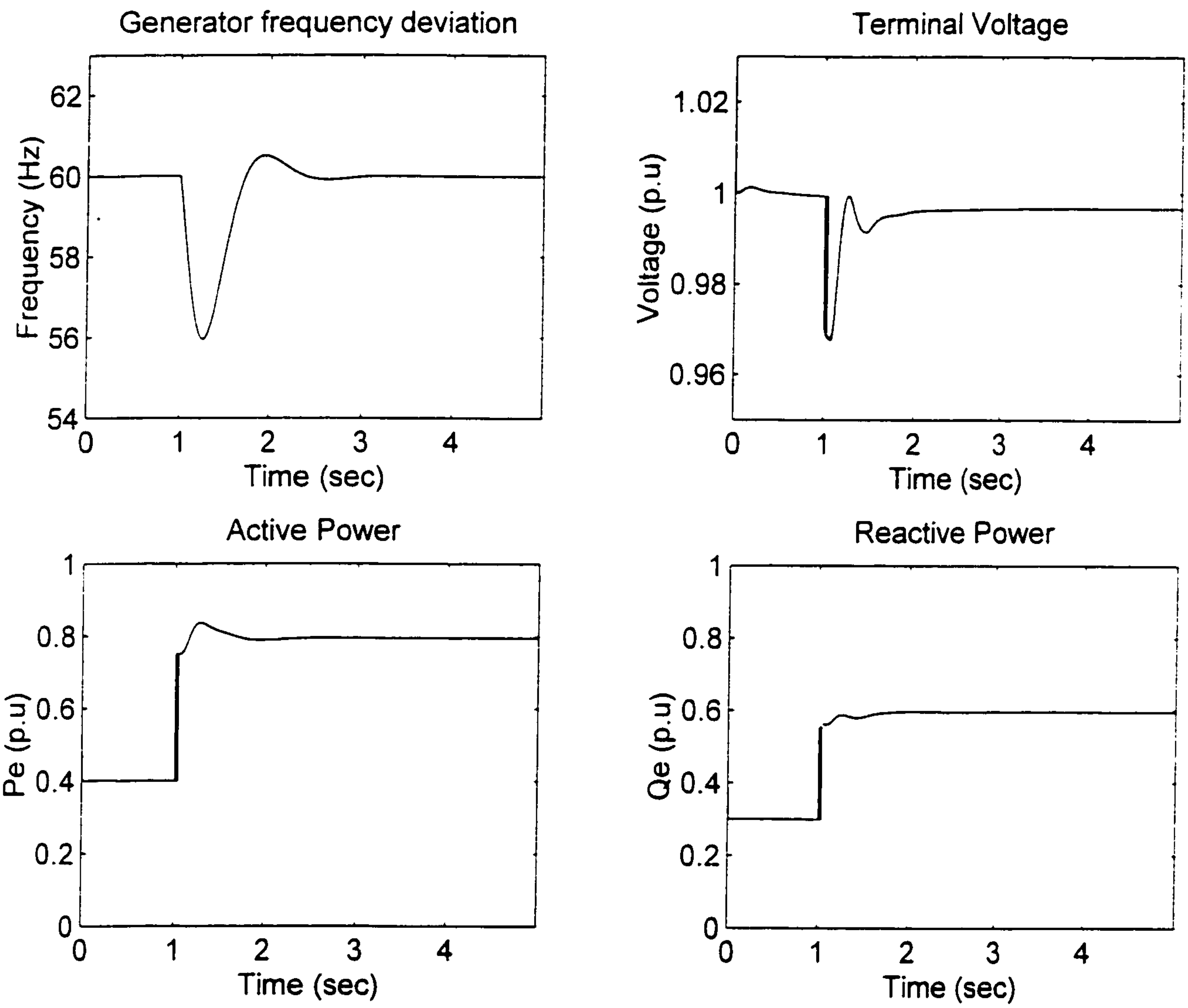

Fig 2.10 Response to step increase of 0.4+j0.3 pu of load power P_L+jQ_L

2.5.2 Step changes in frequency & voltage settings:

The purpose of this to see the effect of changing the input speed (frequency) or voltage settings on the outputs, to verify the validity of the model.

CASE - 1 : Step increase in speed setting ω_{REF}

A 5% step increase is applied to ω_{REF}. The same parameters recorded in section 2.5.1 are plotted again and shown in Fig 2.11. It is noticed that the frequency increased 3 Hz (5%) while the voltage and the active power were slightly disturbed and no affect on the reactive power.

CASE - 2 : Step Increase in voltage setting V_{REF}

A 5% step increase is applied to V_{REF}. The same parameters recorded in section 2.5.1 are plotted again and shown in Fig 2.12. It is noticed that the voltage increased to 1.045 p.u and the frequency was slightly disturbed by 0.5 Hz dip before it settles to 60Hz. The active and reactive power increased about 0.04 p.u. This increase is caused by the disturbance of the internal voltage Eq' which is directly affected by the excitation loops. The slight increase in the active power caused the frequency to dip 0.5 Hz.

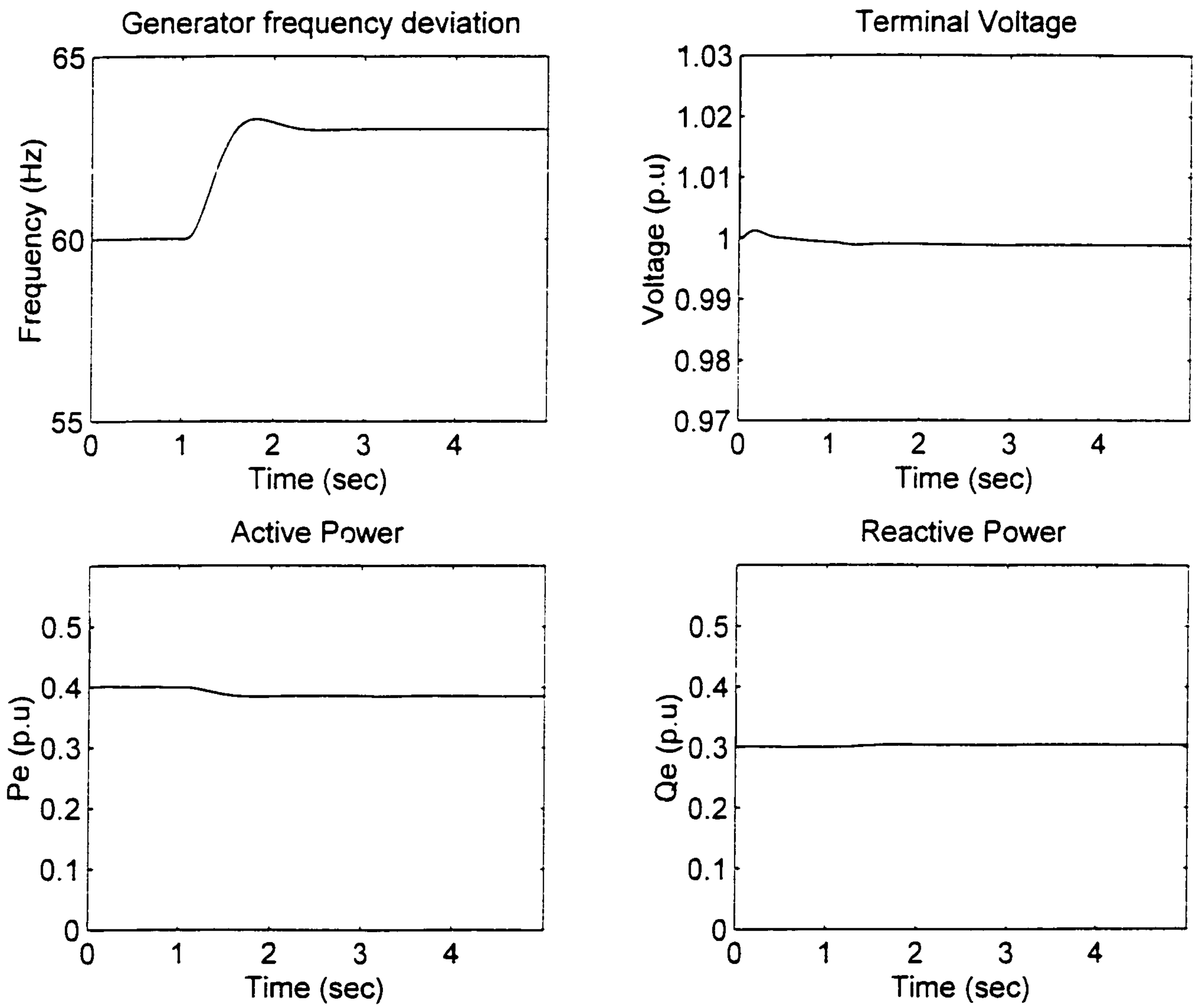

Fig 2.11 Response to step increase of 0.05 pu in speed reference ω_{REF}

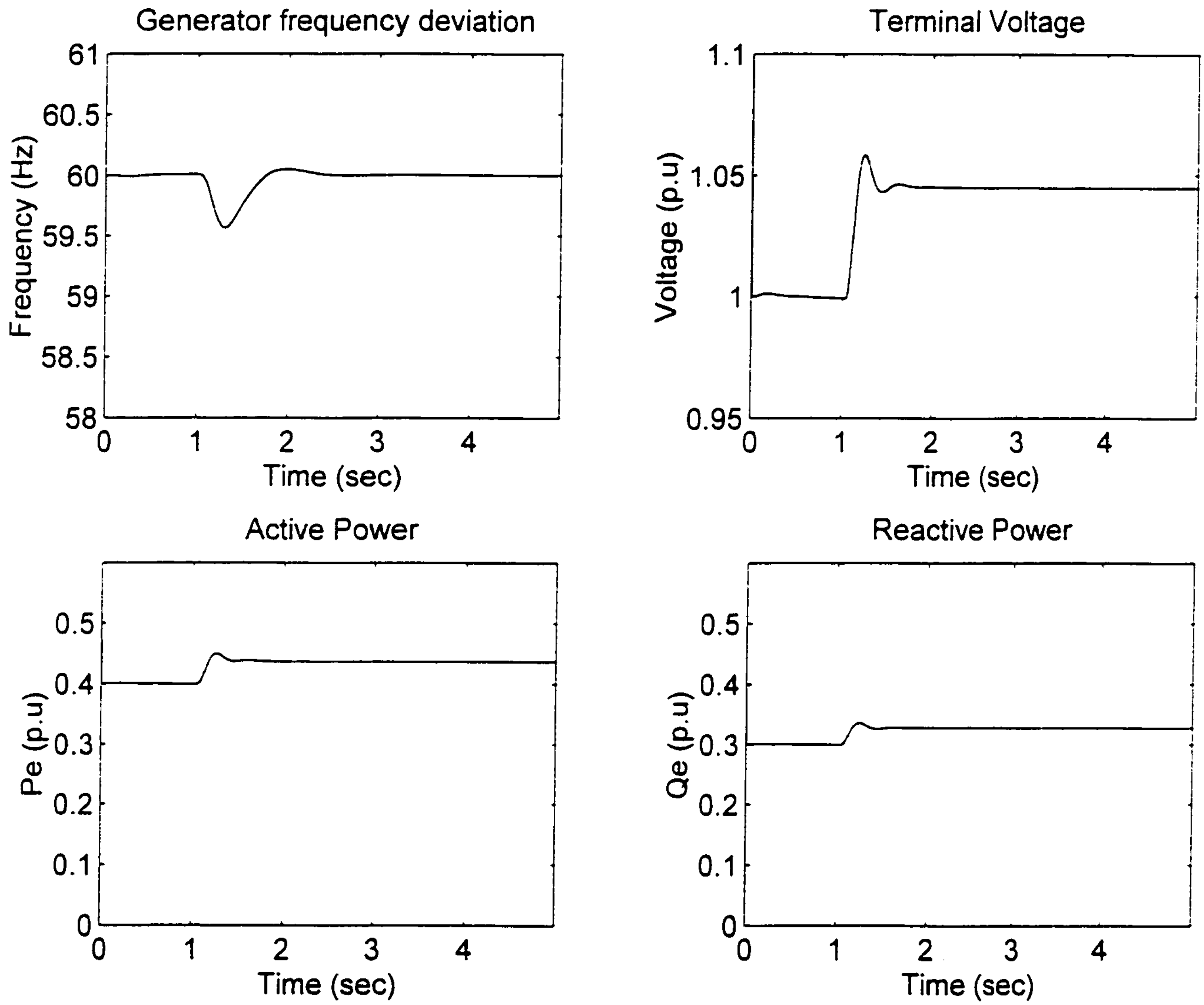

Fig 2.12 Response to step increase of 0.05 pu in voltage reference V_{REF}

2.7.5 Generator set real test records:

A real step load tesdt was carried out on the generator desinated by DG-1. The complete generator's parameters are detailed in Appendix-A. The test was carried out in the premises of Saudi Diesel Generators Company in Al-Khobar, Saudi Arabia. The transient frequency deviation of the generator was obtained and recorded. Fig 2.13 shows the generators set under test, where Fig 2.14 shows the testing instrument setup using *Gould Viper-TA ™* chart recorder. The response is plotted in Fig 2.15(a). The test is done into two steps. First step of 0.6 p.u (60% of its rated power) was applied on the generator and the frequency dip was recorded to be 5 Hz. The speed of the generator recovered in about 3 seconds. To compare this response with the simulation test, the simulation test was repeated for the same step increase of 60% (0.48 p.u. on DG-1 base). The result is shown in Fig 2.15(b) which gives suimilar frequency dip of 5 Hz and smoother recovery in 2 seconds. The difference in the smoothness of the response as compared to the real test is expected because the engine model used in the simulation study did not take into account diesel engine nonlinearities and other engine' restrictions such as air inlet and turbocharger effect. For turbo-charged engines the engine will pickup usually after the charger build up the necessary air pressure to the combustion chambers. This may vary from one engine to another. Another reason for the difference from the simulation study is the speed controller gain settings. In the simulation study the gain constants were selected and fixed after several trials to get the best response. While in actual testing the controller performance is affected not only by its settings but also by its DC power supply, and the response of the actuator to its signals. In spite of all of the above the real test proved that the model represents the system with high accuracy.

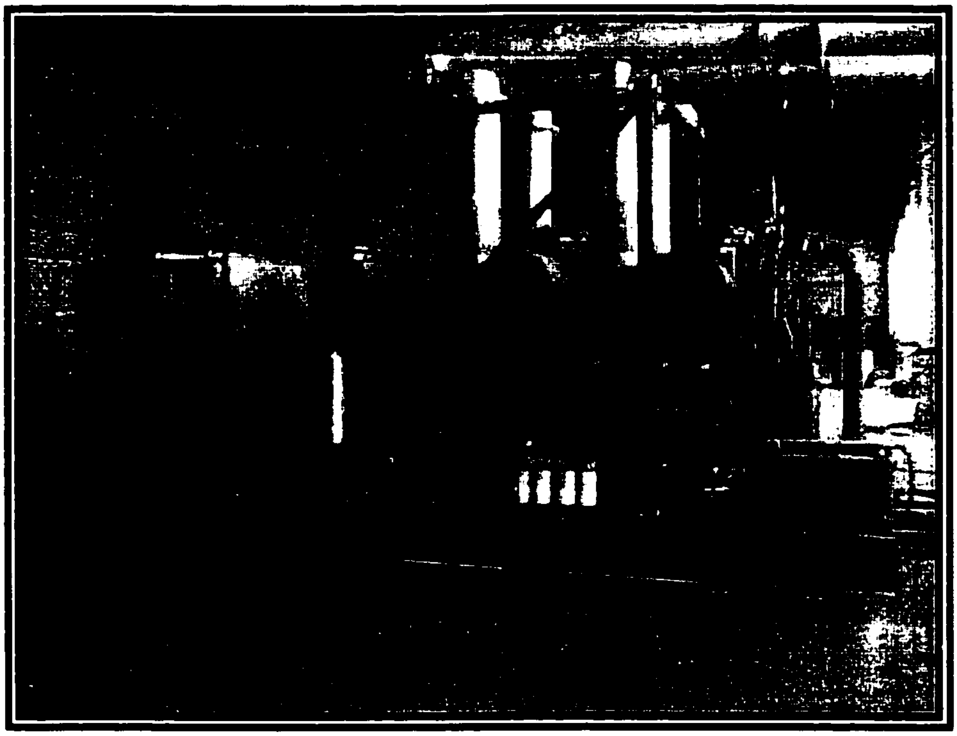

Fig 2.13 Generator set during test

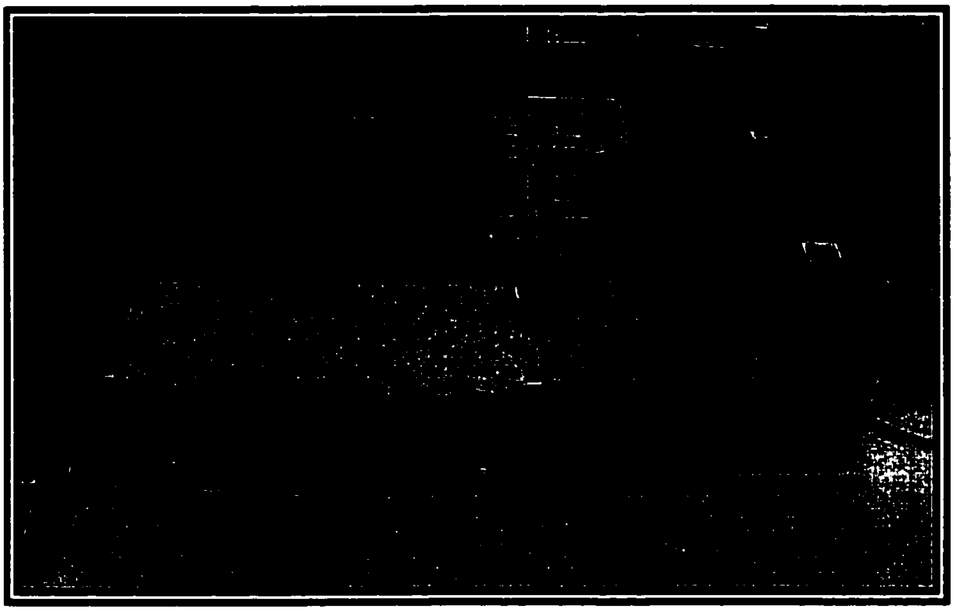

Fig 2.14 Gould Viper-TA™ Chart Recorder

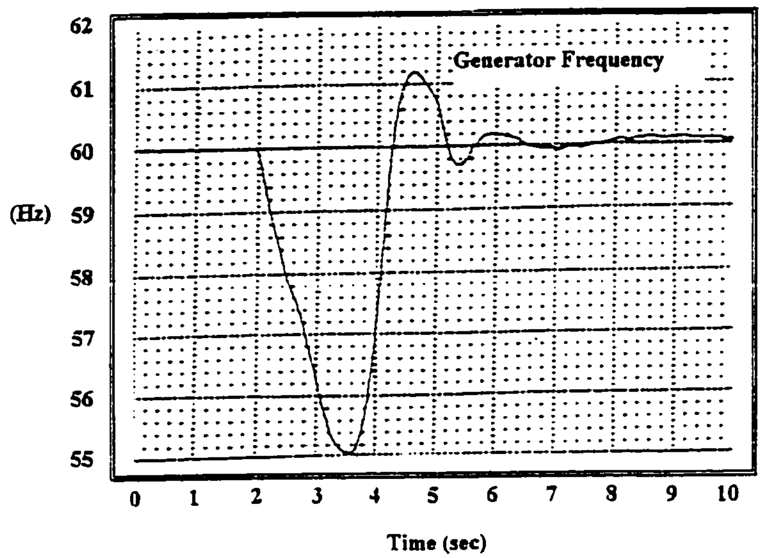

Fig 2.15(a) Real generator's frequency deviation records for a step load in P_L 0-60% step resistive load was applied on the generator.

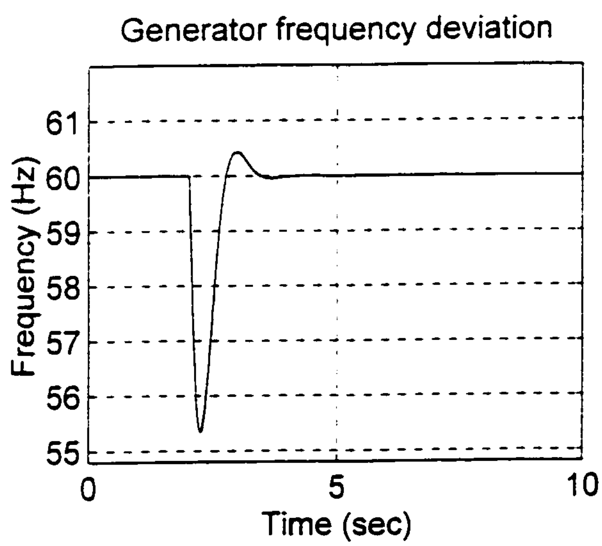

Fig 2.15(b) Simulation test response to a step load of 60% in P_L (0.48 p.u on DG-1 base)

2.6 Linearization of Generator Model:

The set of nonlinear dynamic equations (2.1), (2.2), (2,11), (2.13), (2.14), (2.15), (2.23), (2.25) and (2.27) are linearized around the initial operating condition subscripted by o. The purpose of linearizing the models is to help design suitable controller. However, when a controller is designed using a linearized version of a model then it should be tested on the nonlinear model before a conclusion is made. These can be written as

$$\dot{x} = Ax + Bu \qquad , \qquad y = Cx + Du \tag{2.29}$$

Where the A and B matrices are given in equations (2.30) and (2.31). The state vector X is $[\Delta\omega, \Delta\delta, \Delta E_q', \Delta E_{fd}, \Delta V_R, \Delta V_F, \Delta X_A, \Delta X_G, \Delta P_m]^T$. The control vector $U = [\omega_{REF}, V_{REF}, P_L]^T$.

$$A = \begin{vmatrix} -D/M & 0 & -2K_{po}E_q{'}_o/M & 0 & 0 & 0 & 0 & 0 & 1/M \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_{qo}/\tau_{do}' & 1/\tau_{do}' & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(K_E+S_E)/T_E & 1/T_E & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_AK_{Vo}/T_A & 0 & -1/T_A & -K_A/T_A & 0 & 0 & 0 \\ 0 & 0 & -K_AK_FK_{Vo}/T_AT_F & 0 & -K_F/T_AT_F & -(K_AK_F+T_A)/T_AT_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/T_2 & K_2K_3/T_2 & 0 \\ d_1 & 0 & d_2 & d_3 & 0 & 0 & -d_6 & d_7 & d_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_1/T_2+2K_1/T_1 & -K_1K_2K_3/T_2 & -2/T_1 \end{vmatrix} \tag{2.30}$$

$$B = \begin{vmatrix} 0 & 0 & E_q{'}_o\Delta K_p \\ 0 & 0 & 0 \\ 0 & 0 & -K_{qo}\,\Delta K_q/\tau_{do}' \\ 0 & 0 & 0 \\ 0 & K_A/T_A & -K_AE_q{'}_o\,\Delta K_V/T_A \\ 0 & K_AK_F/T_AT_F & -K_AK_FE_q{'}_o\Delta K_V/T_AT_F \\ 0 & 0 & 0 \\ K_{IN} & 0 & \Delta K_W \\ 0 & 0 & 0 \end{vmatrix} \tag{2.31}$$

d_1, d_2, d_3, d_6, d_7, d_8 are all constants defined similar way to m_1 to m_7 in the nonlinear case. These are defined in Appendix B.2. Also we define ΔK_W, ΔK_P, ΔK_Q and ΔK_q as the generator load change incremental constants. These are defined in Appendix-B.2.

The outputs (Y vector) are considered to be ω, V_t, P_e and Q_e, which are related to the states as follows

$$\begin{vmatrix} \Delta\omega \\ \Delta V_t \\ \Delta P_e \\ \Delta Q_e \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{Vo} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2K_{Po}E'_{qo} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2K_{Qo}E'_{qo} & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \Delta\omega \\ \Delta\delta \\ \Delta E_q' \\ \Delta E_{fd} \\ \Delta V_R \\ \Delta V_F \\ \Delta X_A \\ \Delta X_G \\ \Delta P_m \end{vmatrix} \tag{2.32}$$

$D_1 = [0\ \ 0\ \ 0]$ $\quad D_2 = [0\ \ 0\ \ \Delta K_V]$ $\quad D_3 = [0\ \ 0\ \ \Delta K_P]$

$D_4 = [0\ 0\ \Delta K_Q]$ $\quad D_5 = [0\ 0\ 0]$ $\quad D_6 = [0\ 0\ 0]$

The composite model for the complete stand-alone diesel generator is shown in Fig 2.16.

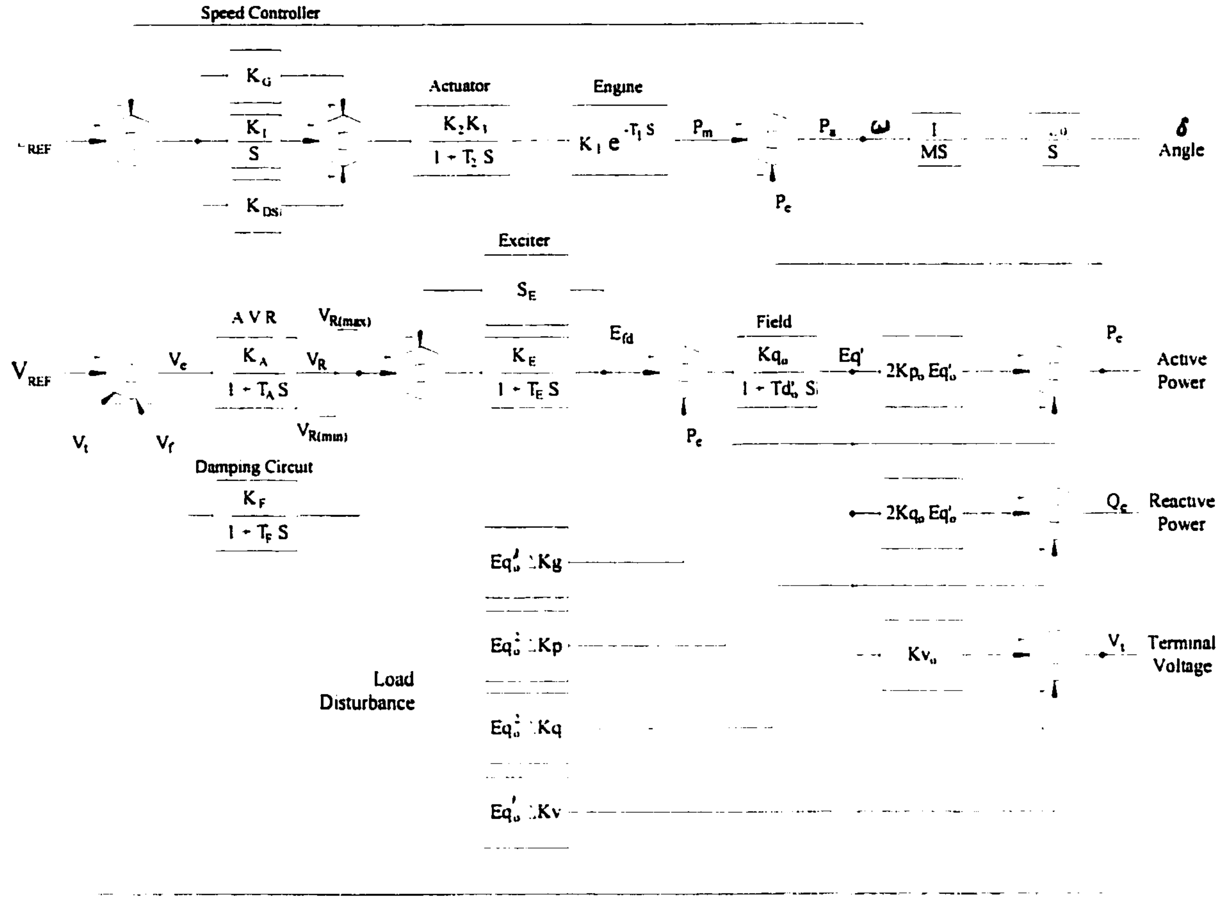

Fig 2.16 Composite Linear Model Representation of stand-alone diesel generator

2.7 Simulation Study of the Linearized Model:

The eigen values of the A matrix given in equation (2.30) are

- 1.04

- 3.02 + j8.52

- 3.02 - j8.52

- 3.77 + j3.78

- 3.77 - j3.78

- 9.58 + j15.54

- 9.58 - j15.54

- 176.36

which indicates the stability for the linearized system.

For comparison purposes, the linear system is tested by applying a step increase in P_L the same way as in the nonlinear case, then it tested by step increase of its speed reference ω_{REF} and V_{REF}.

A step increase of 0.4 p.u of the load power P_L is given. The plots for the response of frequency and rotor angle are shown in Fig 2.17 while the plots of the terminal voltage and the electrical power are shown in Fig 2.18.

Then a 5% step increase in the speed reference ω_{REF} is given and the comparative response for frequency is plotted in Fig 2.19. While Fig 2.20 shows the response of the terminal voltage to a 5% step increase in V_{REF}.

It is noticed that the linearized model gives very closed response to the nonlinear one, which means the linearized model can be used for simulation studies and controller design.

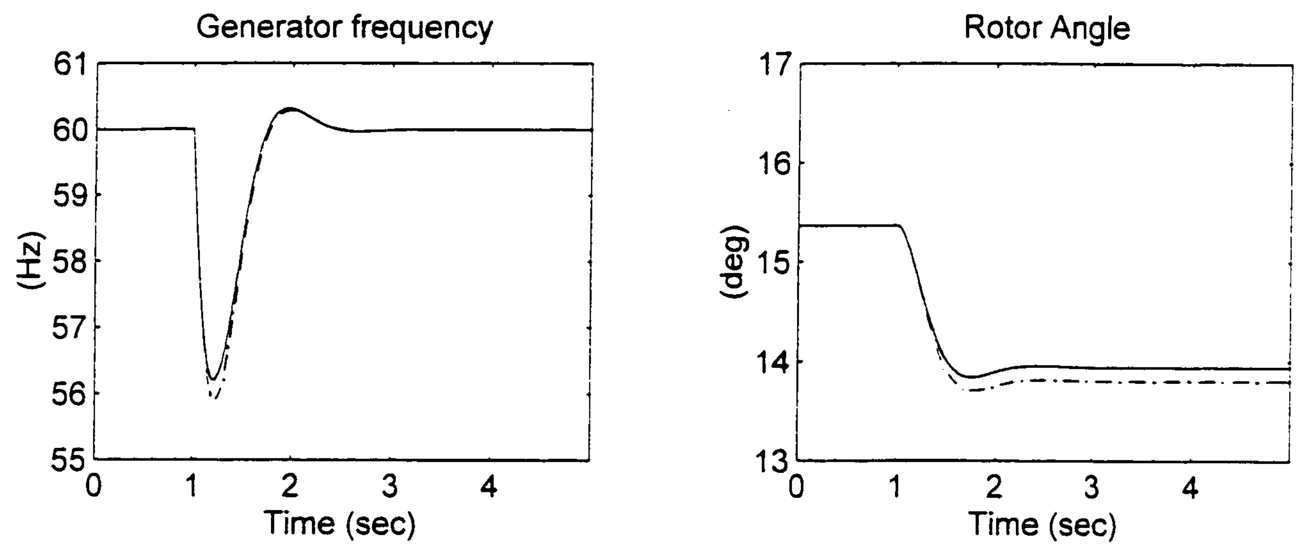

Fig 2.17 Frequency and rotor angle response to step increase of 0.4 pu of active load power P_L ------ Linear —— Nonlinear

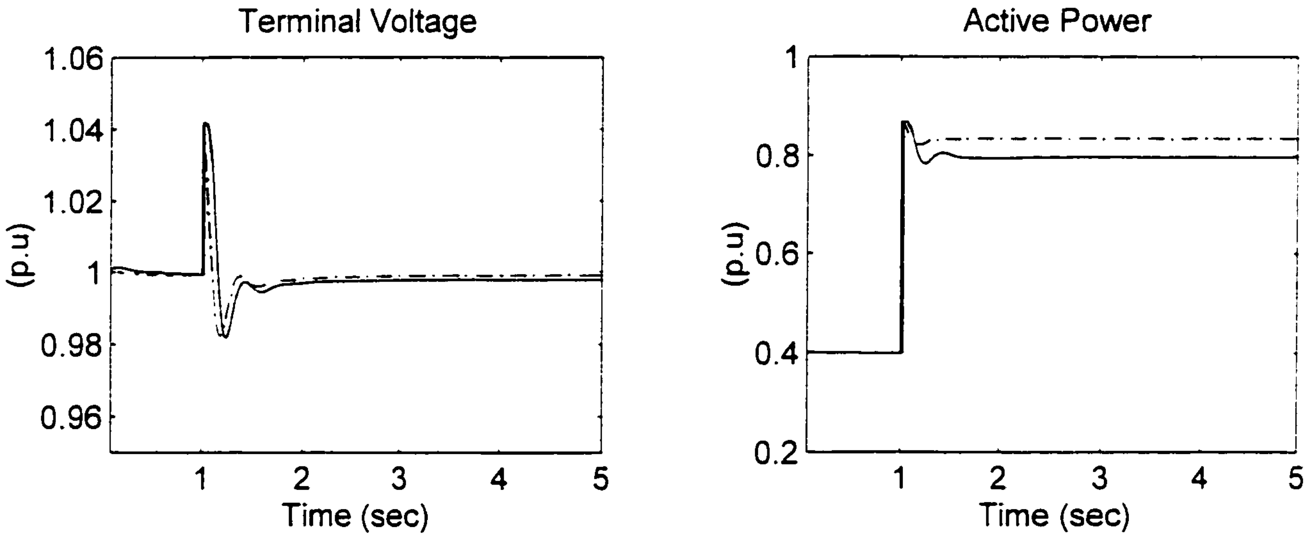

Fig 2.18 Terminal voltage and electrical power response to step increase of 0.4 pu of active load power P_L ------ Linear —— Nonlinear

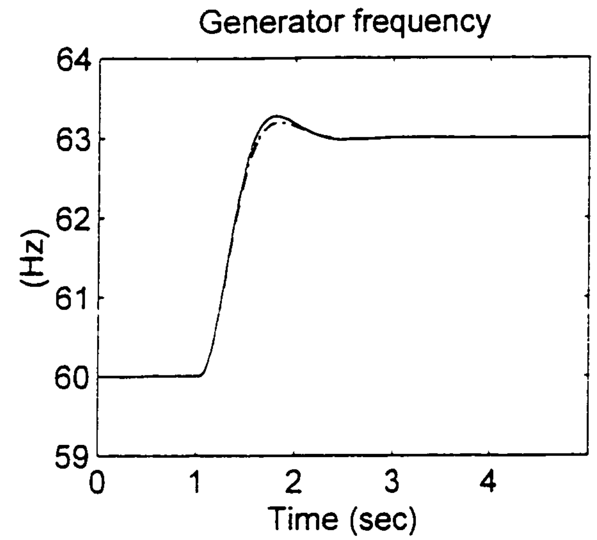

Fig 2.19 Frequency response to step increase of 5% in ω_{REF}

------ Linear ____ Nonlinear

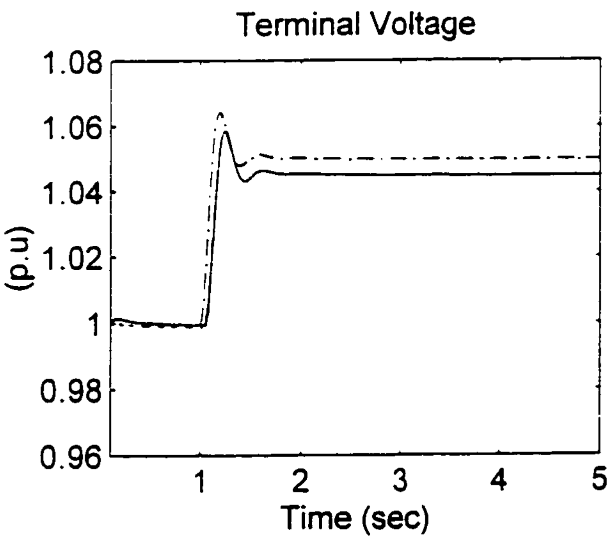

Fig 2.20 Terminal voltage response to step increase of 5% in V_{REF}

------ Linear ____ Nonlinear

Chapter 3

MODELING OF PARALLEL DIESEL GENERATORS

In chapter-2 we have developed a complete model for a stand-alone diesel generator. In this chapter modeling of two diesel generators connected in parallel to a common bus bar and supplying a common local load is considered as shown in Fig 3.1

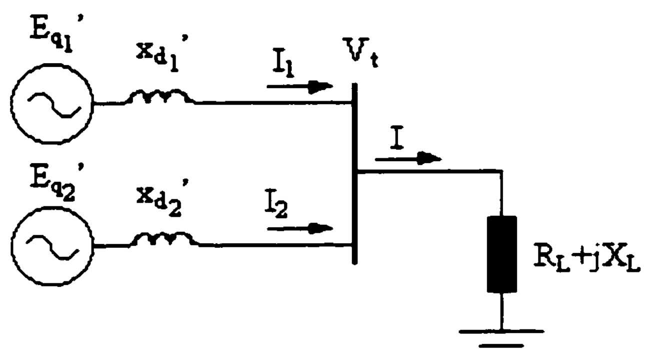

Fig 3.1 Two generators in parallel connected to a local Load

Fig 3.2 shows the main functional control parts of two diesel generators in parallel. The two generator controls interact with each through the main bus-bar via their internal voltages E_q' and internal impedances.

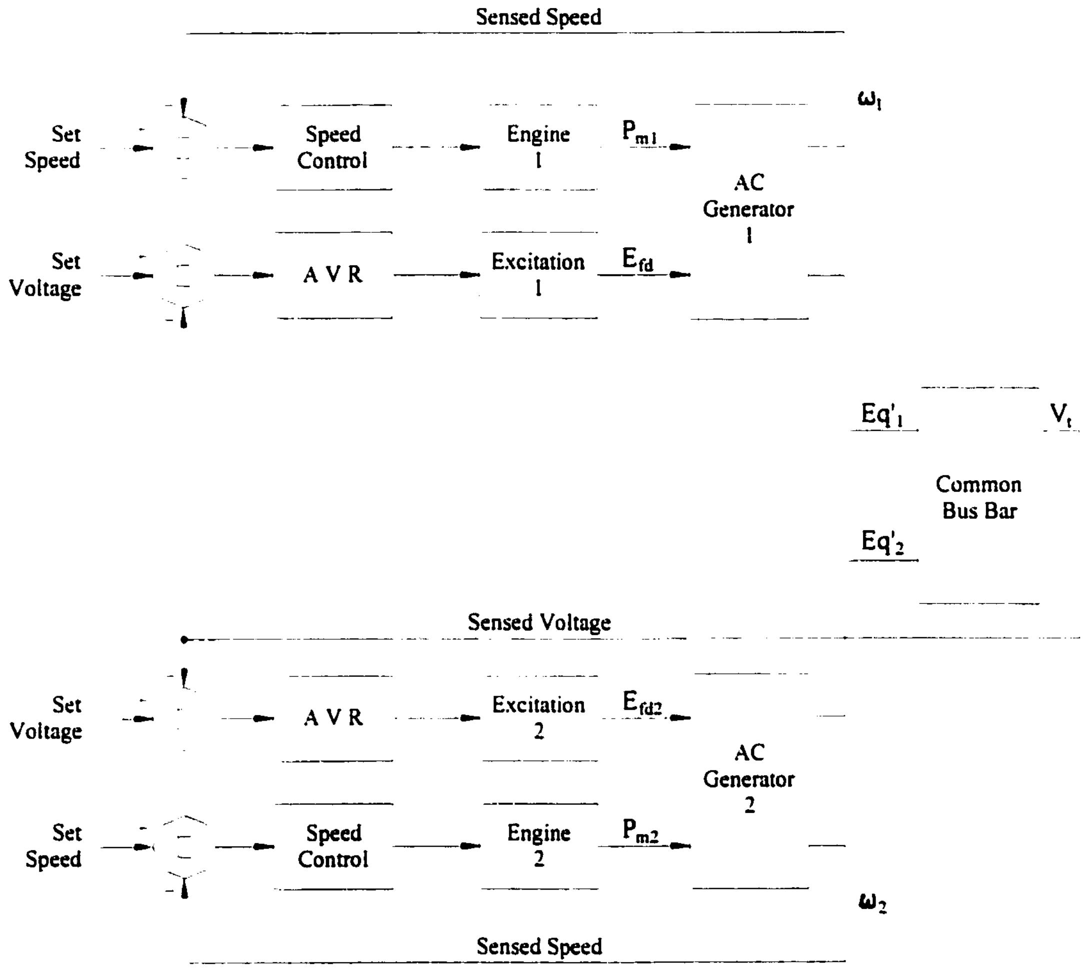

Fig 3.2 Functional control block diagram of two parallel diesel generators

3.1 Two Synchronous Generators Model:

The swing equations for the two synchronous generators are are:

$$\dot{\omega}_1 = (1/M_1)[P_{m_1} - P_{e_1} - P_{D_1}] \tag{3.1}$$

$$\dot{\delta}_1 = \omega_o[\omega_1 - 1] \tag{3.2}$$

$$\dot{\omega}_2 = (1/M_2)[P_{m_2} - P_{e_2} - P_{D_2}] \tag{3.3}$$

$$\dot{\delta}_2 = \omega_o[\omega_2 - 1] \tag{3.4}$$

where P_{e_1} and P_{e_2} are obtained from

$$P_{e_1} = V_d I_{d_1} + V_q I_{q_1} \tag{3.5}$$

$$P_{e_2} = V_d I_{d_2} + V_q I_{q_2} \tag{3.6}$$

From the relation

$$V_t = Z_L I \tag{3.7}$$

$$= (R_L + jX_L)[(I_{d_1} + I_{d_2}) + j(I_{q_1} + I_{q_2})] \tag{3.8}$$

$$= V_d + jV_q \tag{3.9}$$

we can write

$$V_d = R_L(I_{d_1} + I_{d_2}) - X_L(I_{q_1} + I_{q_2}) \tag{3.10}$$

$$V_q = X_L(I_{d_1} + I_{d_2}) + R_L(I_{q_1} + I_{q_2}) \tag{3.11}$$

But we also can write V_d and V_q as

$$V_d = x_{q_1} I_{q_1} \tag{3.12}$$

$$= x_{q_2} I_{q_2} \tag{3.13}$$

$$V_q = E_{q_1}' - x_{d_1}' I_{d_1} \tag{3.14}$$

$$= E_{q_2}' - x_{d_2}' I_{d_2} \tag{3.15}$$

$$I_{q_2} = (x_{q_1}/x_{q_2})I_{q_1} \tag{3.16}$$

$$I_{d_2} = (E_{q_2}'-E_{q_1}'+x_{d_1}'I_{d_1})/x_{d_2}' \tag{3.17}$$

From which we write I_{d_1} , I_{d_2} , I_{q_1} , I_{q_2} , V_d and V_q in terms of machine internal voltages E_{q_1}' and E_{q_2}'.

$$I_{d1} = X_1E_{q_1}'-X_{10}E_{q_2}' \tag{3.18}$$

$$I_{q1} = X_{11}E_{q_1}'+X_{12}E_{q_2}' \tag{3.19}$$

$$I_{q2} = X_{13}E_{q_1}'+X_{14}E_{q_2}' \tag{3.20}$$

$$I_{d2} = X_{15}E_{q_1}'+X_{16}E_{q_2}' \tag{3.21}$$

$$V_d = X_{17}E_{q_1}'+X_{18}E_{q_2}' \tag{3.22}$$

$$V_q = X_{19}E_{q_1}'+X_{20}E_{q_2}' \tag{3.23}$$

X_1 to X_{20} are system reactance constants. These are functions of the two machine reactances as defined in Appendix-C.2.

The internal field voltage equations for the two generators are

$$\dot{E}_{q_1}' = (1/\tau_{do_1}')(E_{fd_1}+X_{27}E_{q_1}'+X_{28}E_{q_2}') \tag{3.24}$$

$$\dot{E}_{q_2}' = (1/\tau_{do_2}')(E_{fd_2}+X_{29}E_{q_1}'+X_{30}E_{q_2}') \tag{3.25}$$

It is noticed that the internal field voltage of each generator is affected by the internal field voltage of the other generator. This is coming from the parallel operation, where the individual generator is no longer a function of its own parameters only but also other geenrators parameters.

3.2 Excitation System Model:

As in the single machine case the excitation system equations for generator # 1 are

$$\dot{V}_{R_1} = -(K_{A_1}V_t)/T_{A_1} - V_{R_1}/T_{A_1} - (K_{A_1}V_{F_1})/T_{A_1} + (K_{A_1}V_{REF_1})/T_{A_1} \tag{3.26}$$

$$\dot{E}_{fd_1} = V_{R_1}/T_{E_1} - (K_{E_1}+S_{E_1})E_{fd_1}/T_{E_1} \tag{3.27}$$

$$\dot{V}_{F_1} = -(K_{A_1}K_{F_1})V_t/(T_{A_1}T_{F_1}) - (K_{A_1}K_{F_1}+T_{A_1})V_{F_1}/(T_{A_1}T_{F_1}) + K_{A_1}K_{F_1}V_{REF_1}/(T_{A_1}T_{F_1}) - K_{F_1}V_{R_1}/(T_{A_1}T_{F_1}) \tag{3.28}$$

Similarly for generator # 2 we can write

$$\dot{V}_{R_2} = -(K_{A_2}V_t)/T_{A_2} - V_{R_2}/T_{A_2} - (K_{A_2}V_{F_2})/T_{A_2} + (K_{A_2}V_{REF_2})/T_{A_2} \tag{3.29}$$

$$\dot{E}_{fd_2} = V_{R_2}/T_{E_2} - (K_{E_2}+S_{E_2})E_{fd_2}/T_{E_2} \tag{3.30}$$

$$\dot{V}_{F_2} = -(K_{A_2}K_{F_2})V_t/(T_{A_2}T_{F_2}) - (K_{A_2}K_{F_2}+T_{A_2})V_{F_2}/(T_{A_2}T_{F_2}) + K_{A_2}K_{F_2}V_{REF_2}/(T_{A_2}T_{F_2}) - K_{F_2}V_{R_2}/(T_{A_2}T_{F_2}) \tag{3.31}$$

where the terminal voltage $V_t = \sqrt{V_d^2 + V_q^2}$ is written in terms of $E_q'_1$ and $E_q'_2$ as,

$$V_t = \sqrt{(X_{17}E_{q_1}'+X_{18}E_{q_2}')^2+(X_{19}E_{q_1}'+X_{20}E_{q_2}')^2} \tag{3.32}$$

3.3 Diesel Engine & Speed Control Equations:

The dynamic equations for the diesel engine and speed control states are identical for actuator state X_A and mechanical power P_m. From (2.23) and (2.25) we can write for generator # 1 and 2 as follows:

$$\dot{X}_{A_1} = -(1/\tau_{2_1})X_{A_1} + (K_{2_1}K_{3_1}/\tau_{2_1})X_{G_1} \tag{3.33}$$

$$\dot{X}_{A_2} = -(1/\tau_{2_2})X_{A_2} + (K_{2_2}K_{3_2}/\tau_{2_2})X_{G_2} \tag{3.34}$$

Therefore mechanical powers are written as,

$$\dot{P}_{m_1} = -(2/\tau_{1_1})P_{m_1} + [K_{1_1}/1\tau_{2_1} + 2K_{1_1}/\tau_{1_1}]\, X_{A_1} - (K_{1_1}K_{2_1}K_{3_1}/\tau_{2_1})\, X_{G_1} \tag{3.35}$$

$$\dot{P}_{m_2} = -(2/\tau_{1_2})P_{m_2} + [K_{1_2}/1\tau_{2_2} + 2K_{1_2}/\tau_{1_2}]\, X_{A_2} - (K_{1_2}K_{2_2}K_{3_2}/\tau_{2_2})\, X_{G_2} \tag{3.36}$$

The dynamic equation of the speed Controller State X_G will be affected by the swing equation electrical power terms (P_{e_1} and P_{e_2}). These are the links between the two generators. This can be seen clearly in the non-linear relation between X_{G_1} or X_{G_2} from one side, and E_{q_1}', E_{q_2}', E_{fd_1} and E_{fd_2} from the other side. This is found to be

$$\dot{X}_{G_1} = m_{1_1}\omega_1 + m_{2_1}P_{e_1} + m_{3_1}P_{m_1} + m_{4_1}\dot{P}_{e_1} - m_{5_1}X_{A_1} + m_{6_1}X_{G_1} + K_{IN_1}\omega_{REF_1} \tag{3.37}$$

$$\dot{X}_{G_2} = m_{1_2}\omega_2 + m_{2_2}P_{e_2} + m_{3_2}P_{m_2} + m_{42}\dot{P}_{e_2} - m_{5_2}X_{A_2} + m_{6_2}X_{G_1} + K_{IN_2}\omega_{REF_2} \tag{3.38}$$

3.4 Complete Model:

The system can be expressed by combining equations (3.1), (3.2), (3.3), (3.4), (3.24), (3.25), (3.26), (3.27), (3.28), (3.29), (3.30), (3.31), (3.33), (3.34), (3.35), (3.36), (3.37) and (3.38) in the form

$$\dot{X} = f\,[x,u]$$

Where the states and the input vectors are,

$$X = [\omega_1, \delta_1, E_q{}'_1, E_{fd_1}, V_{R_1}, V_{F_1}, X_{A_1}, X_{G_1}, P_{m_1}, \omega_2, \delta_2, E_q{}'_2, E_{fd_2}, V_{R_2}, V_{F_2}, X_{A_2}, X_{G_2}, P_{m_2}]^T$$

$$U = [\omega_{REF_1}, V_{REF_1}, \omega_{REF_2}, V_{REF_2}]^T$$

The outputs are ω_1, P_{e_1}, Q_{e_1}, ω_2, P_{e_2}, Q_{e_2}. The detailed equations of P_e and Q_e for the two generators are given in Appendix-C4.

3.5 Simulation Study:

The two parallel diesel generators were simulated on Matlab. The set of nonlinear differential equations were solved using Matlab ODE4 toolbox. The algorithm is detailed in Appendix-C3, and the system data is detailed in Appendix-A. The initial states and loading parameters equations are detailed in Appendix-C.1

3.5.1 Step Load Test:

The step increase is given to the common load of the two parallel generators. Two different steps. The first one is in P_L while the second is in Q_L. Fig 3.3 shows the response of the two generators to 0.73 p.u step incresae in P_L, where the frequency and the active power are plotted. We notice the stability of the system and we notice that Gen-1 power incresaed from 0.32 p.u to 0.64 p.u (0.32 p.u increment which is 32% of its rating) while Gen-2 power increased from 0.4 p.u to 0.82 p.u (0.42 p.u increment which is 32% of its rating referred to its own base).

Fig 3.4 shows the response of the two generators to 0.45 p.u step incresae in Q_L, where frequency and reactive power are plotted. We notice the same as in the P_L case, where the two reactive powers increased the same percentage to each machine's base, while we notice that the two frequencies were slightly disturbed by Q_L change. Gen-1 frequency with 0.3 Hz dip, while Gen-2 with 0.3 Hz rise.

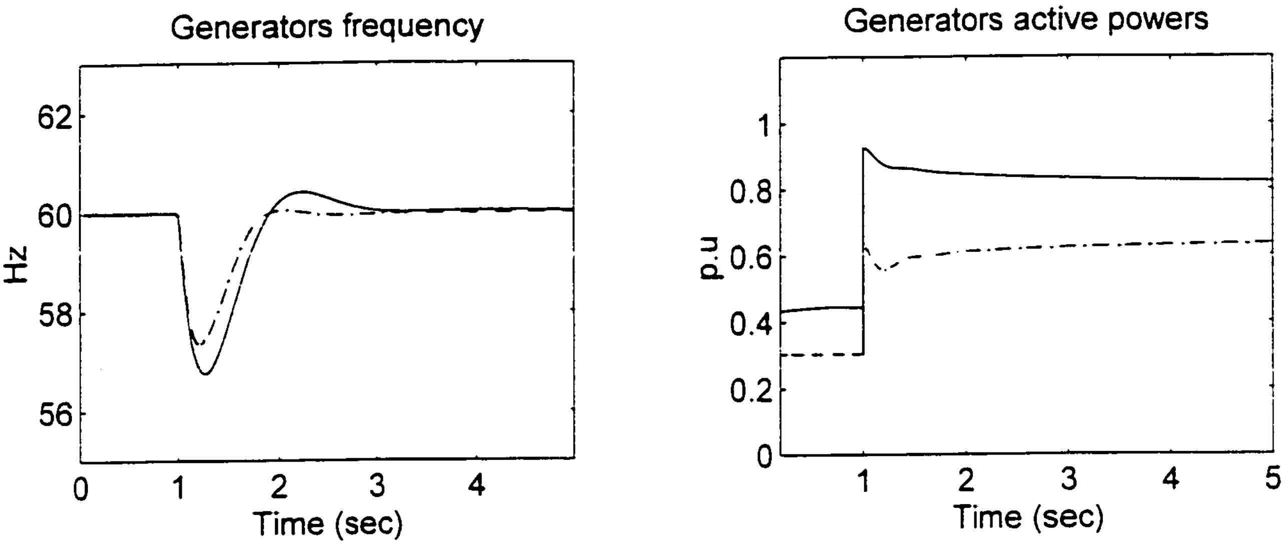

Fig 3.3 Response to a step increase of 0.73 pu of active load power P_L

Frequencis and active powers (- - - gen # 1, —— Gen # 2)

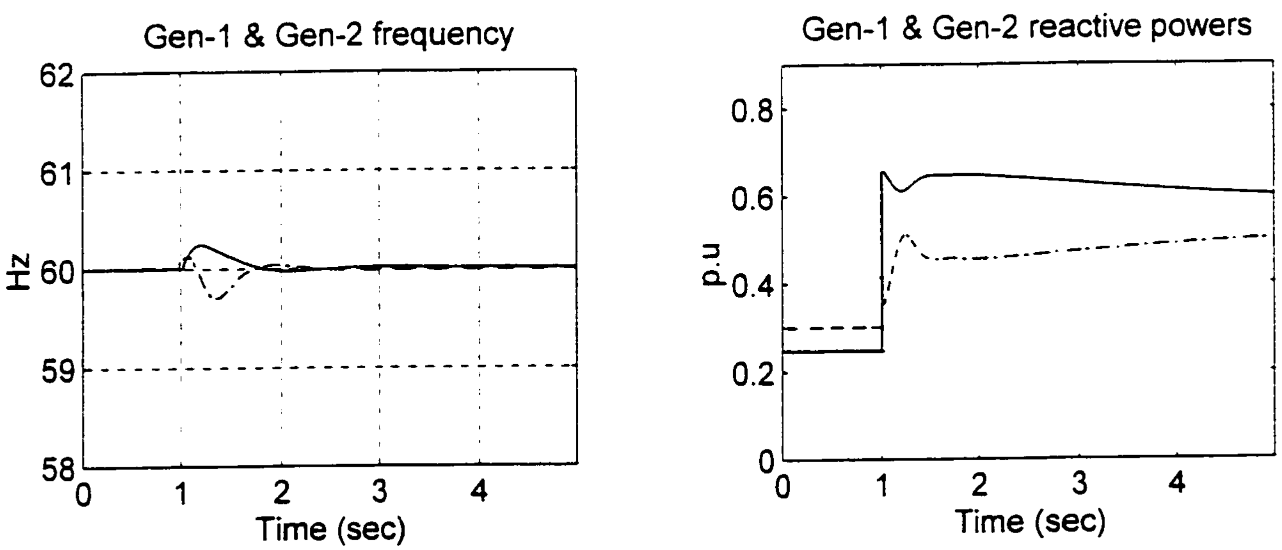

Fig 3.4 Response to a step increase of 0.454 p.u of reactive load power Q_L

Frequencis and reactive powers (- - - gen # 1, —— gen # 2)

3.5.2 Step Increase in Frequency & Voltage Settings:

A 5% step increase is given to speed reference of Gen-1. The two generators frequency deviation responses are plotted in Fig 3.5. We see that the frequency of Gen-1 increases up to approximately 63 Hz which is 5% above 60Hz. Generator # 2 seems not to be affected by this change.

In Fig 3.6 the voltage reference setting V_{REF_2} was given a step increase of 5%. The terminal voltage response was plotted. It is noticed that the terminal voltage increased by less than 3% because the terminal voltage is affected by both generators, and only V_{REF_2} was changed.

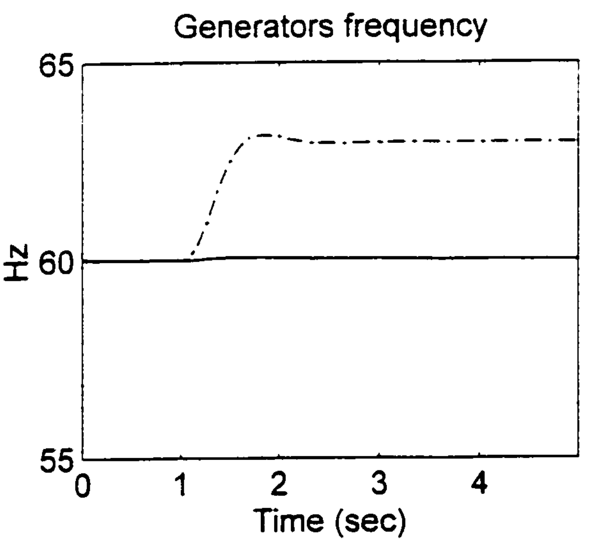

Fig 3.5 Response to a step increase of 5% of speed setting ω_{REF_1} (Two generators frequencies)
--- Gen-1 —— Gen-2

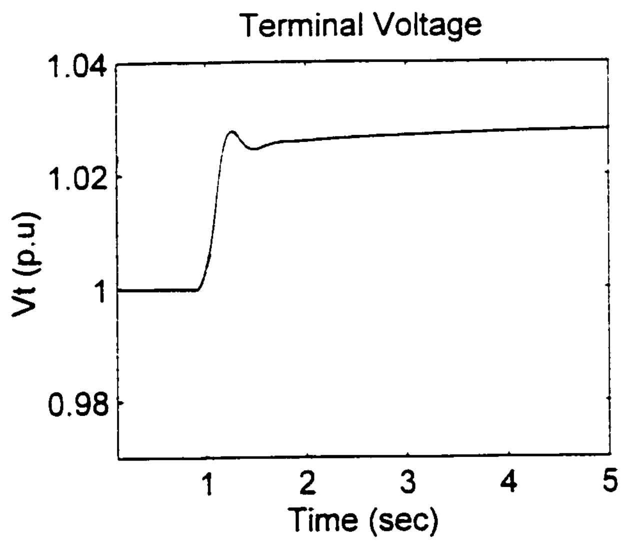

Fig 3.6 Response to a step increase of 5% of voltage setting V_{REF_2} (Terminal voltage)

3.6 Linearization of Parallel Generators Model:

The set of dynamic equations obtained for the two parallel generators (3.1), (3.2), (3.3), (3.4), (3.24), (3.25), (3.26), (3.27), (3.28), (3.29), (3.30), (3.31), (3.33), (3.34), (3.35), (3.36), (3.37) and (3.38) are linearized around their initial operating conditions subscripted by o. These can be written as,

$\Delta \dot{X} = A\Delta X + B\Delta U$, which is written in state space form in equation (3.39). Similarly the outputs equation $\Delta Y = C\Delta X + D\Delta U$, which is written in matrix form in equation (3.40).

The states vector $\Delta X = [\Delta\omega_1, \Delta\delta_1, \Delta E_q{'}_1, \Delta E_{fd_1}, \Delta V_{R_1}, \Delta V_{F_1}, \Delta X_{A_1}, \Delta X_{G_1}, \Delta P_{m_1}, \Delta\omega_2, \Delta\delta_2, \Delta E_q{'}_2, \Delta E_{fd_2}, \Delta V_{R_2}, \Delta V_{F_2}, \Delta X_{A_2}, \Delta X_{G_2}, \Delta P_{m_2}]^T$. The control vector $\Delta U = [\omega_{REF_1}, V_{REF_1}, \omega_{REF_2}, V_{REF_2}]^T$. The outputs vector $\Delta Y = [\Delta\omega_1, \Delta P_{e_1}, \Delta Q_{e_1}, \Delta\omega_2, \Delta P_{e_2}, \Delta Q_{e_2}, \Delta V_t]^T$

$$
\begin{bmatrix}
\Delta\dot{\omega}_1 \\
\Delta\dot{\delta}_1 \\
\Delta\dot{E}_{q1}' \\
\Delta\dot{E}_{fd1} \\
\Delta\dot{V}_{R1} \\
\Delta\dot{V}_{F1} \\
\Delta\dot{X}_{A1} \\
\Delta\dot{X}_{G1} \\
\Delta\dot{P}_{m1} \\
\Delta\dot{\omega}_2 \\
\Delta\dot{\delta}_2 \\
\Delta E_{q2}' \\
\Delta\dot{E}_{fd2} \\
\Delta\dot{V}_{R2} \\
\Delta\dot{V}_{F2} \\
\Delta\dot{X}_{A2} \\
\Delta\dot{X}_{G2} \\
\Delta\dot{P}_{m2}
\end{bmatrix}
=
\begin{bmatrix}
-D/M_1 & 0 & -K_{P11}/M_1 & 0 & 0 & 0 & 0 & 0 & 1/M_1 & 0 & 0 \\
\omega o & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & X_{27i}/Tdo_1' & 1/T_{do1}' & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(K_{E1}+S_{E1})/T_{E1} & 1/T_{E1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -K_{A1}K_{V1}/T_{A1} & 0 & -1/T_{A1} & -K_{A1}/T_{A1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -K_{A1}K_{F1}K_{V1}/(T_{A1}T_{F1}) & -K_{F1}/(T_{A1}T_{F1}) & 0 & -(K_{A1}K_{F1}+T_{A1})/(T_{A1}T_{F1}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1/T_{21} & K_{21}K_{31}/T_{21} & 0 & 0 & 0 \\
d_{11} & 0 & d_{21} & d_{41} & 0 & 0 & d_{61} & d_{71} & d_{81} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & K_{11}/T_{21}+2K_{11}/T_{11} & -K_{11}K_{21}K_{31}/T_{21} & -2/T_{11} & 0 & 0 \\
0 & 0 & -K_{P21}/M_2 & 0 & 0 & 0 & 0 & 0 & 0 & -D/M_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega o & 0 \\
0 & 0 & X_{29i}/T_{do2}' & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -K_{A2}K_{F2}K_{V1}/(T_{A2}T_{F2}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & d_{22} & d_{42} & 0 & 0 & 0 & 0 & 0 & d_{12} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$-K_{P21}/M_2$	0	0	0	0	0	0	$\Delta\omega_1$
0	0	0	0	0	0	0	$\Delta\delta_1$
X_{28i}/T_{do2}'	0	0	0	0	0	0	$\Delta E_{q1}'$
0	0	0	0	0	0	0	ΔE_{fd1}
$-K_{A1}K_{V2}/T_{A1}$	0	0	0	0	0	0	ΔV_{R1}
$-K_{A1}K_{F1}K_{V2}/(T_{A1}T_{F1})$	0	0	0	0	0	0	ΔV_{F1}
0	0	0	0	0	0	0	ΔX_{A1}
d_{31}	d_{51}	0	0	0	0	0	ΔX_{G1}
0	0	0	0	0	0	0	ΔP_{m1}
$-K_{P22}/M_2$	0	0	0	0	0	$1/M_2$	$\Delta\omega_2$
	0	0	0	0	0	0	$\Delta\delta_2$
X_{30i}/T_{do2}'	$1/T_{do2}'$	0	0	0	0	0	$\Delta E_{q2}'$
0	$-(K_{E2}+S_{E2})/T_{E2}$	$1/T_{E2}$	0	0	0	0	ΔE_{fd2}
$-K_{A2}K_{F2}K_{V2}/(T_{A2}T_{F2})$	0	$-K_{F2}/(T_{A2}T_{F2})$	$-(K_{A2}K_{F2}+T_{A2})/(T_{A2}T_{F2})$	0	0	0	ΔV_{R2}
0	0	0	0	0	0	0	ΔV_{F2}
0	0	0	0	$-1/T_{22}$	$K_{22}K_{32}/T_{22}$	0	ΔX_{A2}
d_{32}	d_{52}	0	0	d_{62}	d_{72}	d_{82}	ΔX_{G2}
0	0	0	0	$K_{12}/T_{22}+2K_{11}/T_{11}$	$-K_{12}K_{22}K_{32}/T_{22}$	$-2/T_{12}$	ΔP_{m2}

$K_{P_{11}}$, $K_{P_{12}}$, $K_{P_{21}}$ and $K_{P_{22}}$ are the active power equation coefficients while K_{V_1}, K_{V_2} and ΔK_V are the terminal voltage equation coefficients. These are all defined in Appendix-C.4. X_{1_i} to X_{50_i} are the initial operating values of X_1 to X_{50}. These are the reactance constants as defined as mentioned before in Appendix-C.2.

$$
+\begin{bmatrix}
0 & 0 & 0 & 0 & -\Delta K_{P1} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Delta K_{q1} \\
0 & 0 & 0 & 0 & 0 \\
0 & K_{A1}/T_{A1} & 0 & 0 & 0 \\
0 & K_{A1}K_{F1}/(T_{A1}T_{F1}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
K_{IN1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\Delta K_{P2} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Delta K_{q2} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & K_{A2}/T_{A2} & 0 \\
0 & 0 & 0 & K_{A2}K_{F2}/(T_{A2}T_{F2}) & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_{IN2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta\omega_{REF1} \\
\Delta V_{REF1} \\
\Delta\omega_{REF2} \\
\Delta V_{REF2} \\
\Delta P_L
\end{bmatrix}
\tag{3.39}
$$

$$\begin{bmatrix} \Delta\omega_1 \\ \Delta P_{e1} \\ \Delta Q_{e1} \\ \Delta\omega_2 \\ \Delta P_{e2} \\ \Delta Q_{e2} \\ \Delta V_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{P11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{P12} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{Q11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{Q22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{P21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{P22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{V1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{V2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega_1 \\ \Delta\delta_1 \\ \Delta E_{q1}' \\ \Delta E_{fd1} \\ \Delta V_{R1} \\ \Delta V_{F1} \\ \Delta X_{A1} \\ \Delta X_{G1} \\ \Delta P_{m1} \\ \Delta\omega_2 \\ \Delta\delta_2 \\ \Delta E_{q2}' \\ \Delta E_{fd2} \\ \Delta V_{R2} \\ \Delta V_{F2} \\ \Delta X_{A2} \\ \Delta X_{G2} \\ \Delta P_{m2} \end{bmatrix}$$

Where d_{1_1}, d_{2_1}, d_{4_1}, d_{6_1}, d_{7_1}, d_{8_1}, d_{3_1} and d_{5_1} are all generator # 1 speed controller equation coefficients. Similarly d_{2_2}, d_{4_2}, d_{1_2}, d_{3_2}, d_{5_2}, d_{6_2}, d_{7_2} and d_{8_2} are generator # 2 speed controller equation coefficients. These are all defined in Appendix-C.3

$$+\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta K_{P1} \\ 0 & 0 & 0 & 0 & \Delta K_{Q1} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta K_{P2} \\ 0 & 0 & 0 & 0 & \Delta K_{Q2} \\ 0 & 0 & 0 & 0 & \Delta K_{V} \end{bmatrix} \begin{bmatrix} \Delta\omega_{REF1} \\ \Delta V_{REF1} \\ \Delta\omega_{REF2} \\ \Delta V_{REF2} \\ \Delta P_{L} \end{bmatrix} \tag{3.40}$$

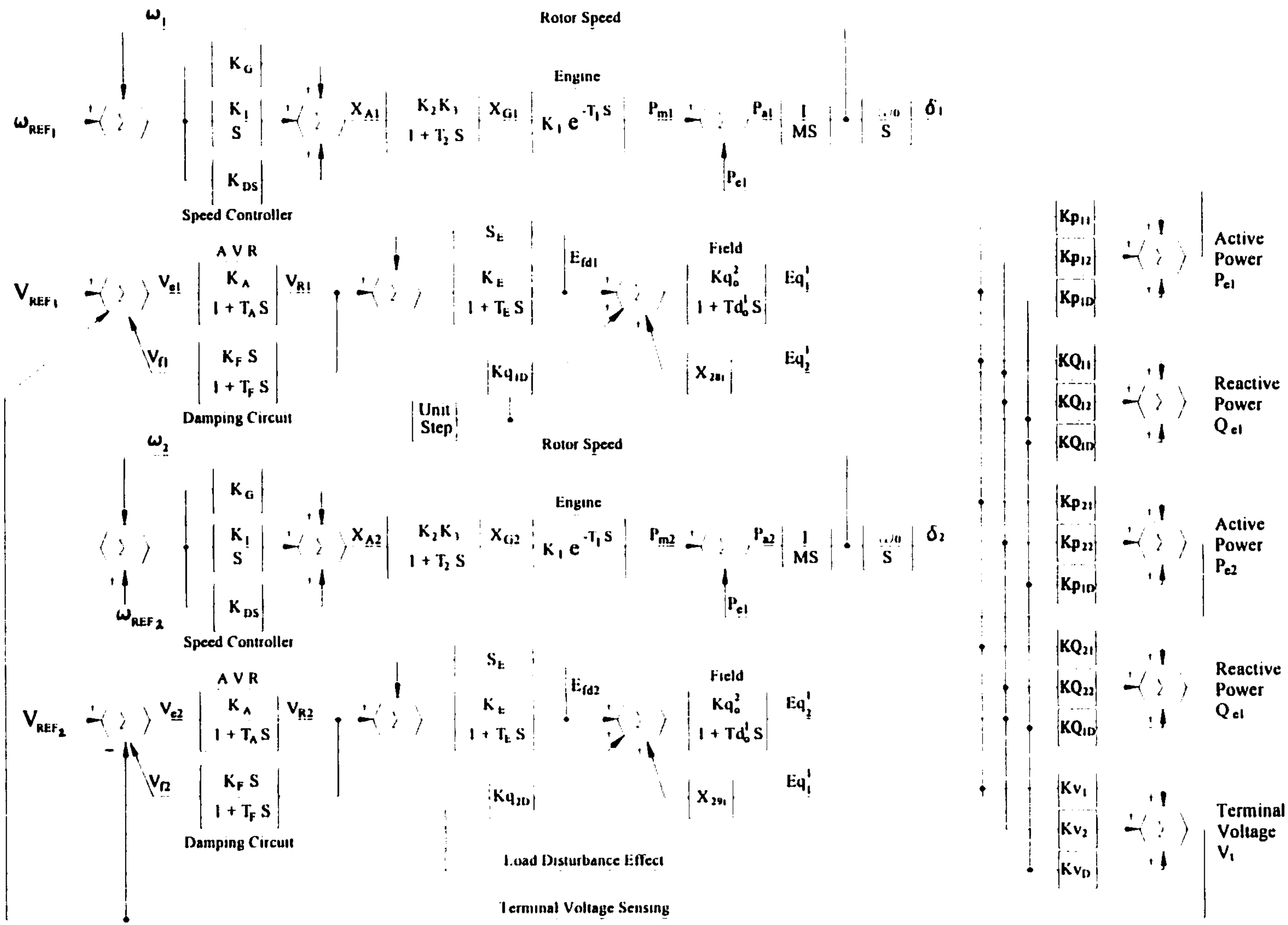

Fig 3.7 Computer model representation of two parallel operating diesel generators

3.7 Simulation Study of the Linearized Model:

The eigen values of the characteristics matrix A of equation 3.39 are

-0.3 , -2.1 ± j0.2i , -3.6 ± j4.3i , -4.3 ± j3.8i , -4.7 ± j13.7i , -6.6 ± j11.1i , -7.0 ± j13.2i , -14.8 , -620.6 , -1059.0

which indicate that the linearized system is stable.

For comparison of the linearized model performance with the nonlinear, the system is tested and and resulting responses are plotted along with the nonlinear ones. The purpose is to show how accurate is the linearized model to represent the system.

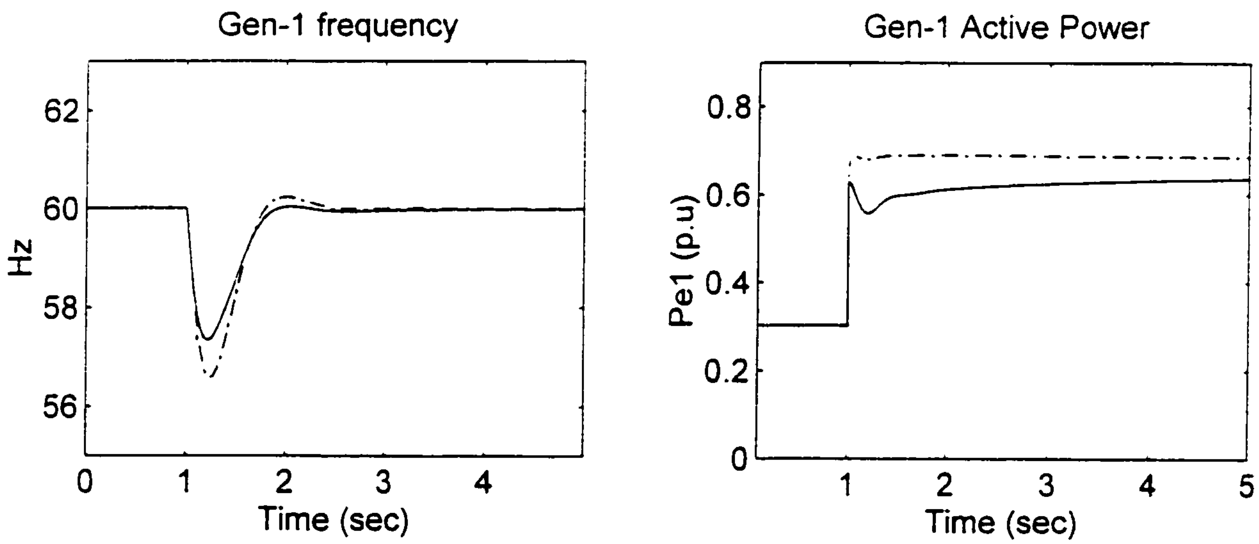

Fig 3.8 Comparative response of Gen-1 frequency and active power to 0.73 p.u step increase in P_L (- - - Linear —— Nonlinear)

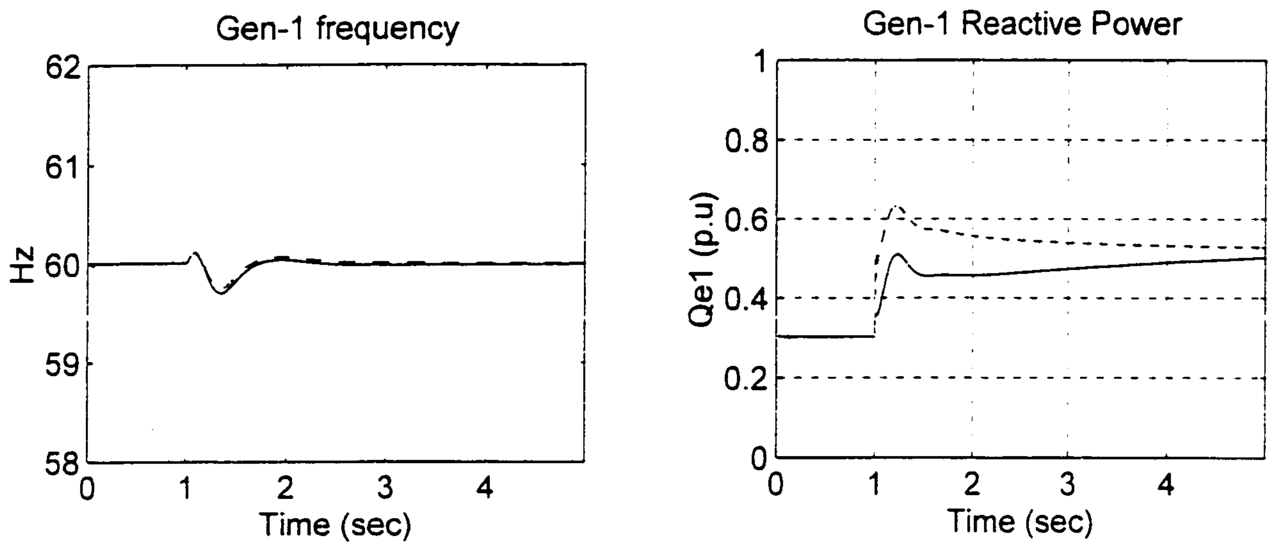

Fig 3.9 Comparative response of Gen-1 frequency and reactive power to 0.45 p.u step increase in Q_L

(- - - Linear —— Nonlinear)

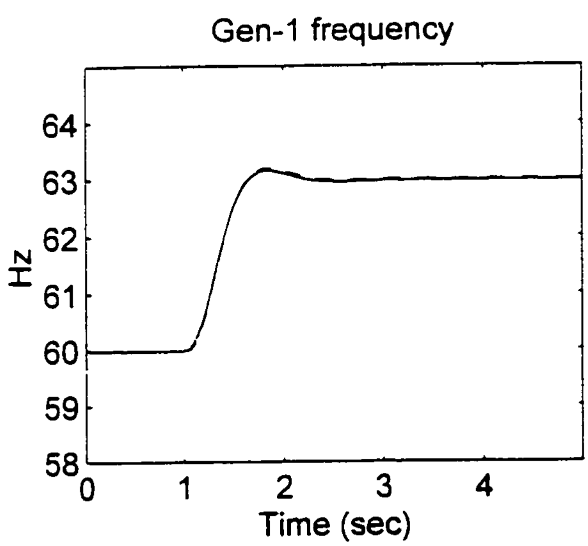

Fig 3.10 Response of generator # 1 frequency to 5% step increase in ω_{REF_1}

(- - - Linear —— Nonlinear)

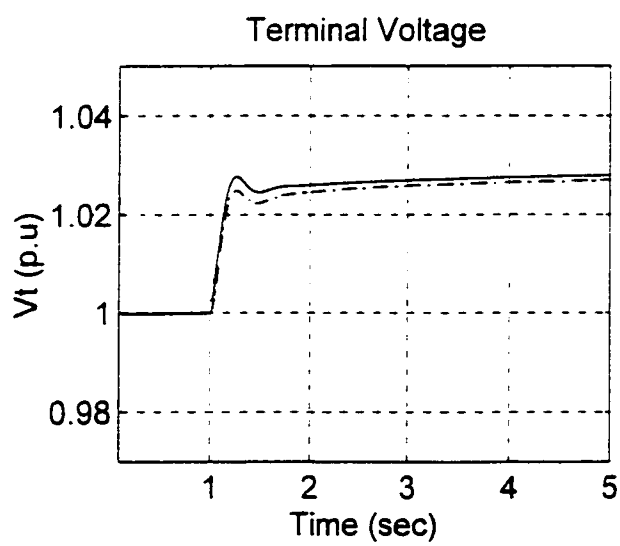

Fig 3.11 Response of terminal voltage to 5% step increase in V_{REF_2}

(- - - Linear —— Nonlinear)

Chapter 4

LOAD SHARING OF PARALLEL DIESEL GENERATORS

When two or more diesel generators operate in parallel, both real and reactive load must be distributed on the generators as percentages of their rated capacities.

It is important to have load-sharing control when diesel generators operate in parallel. This will ensure these generators are loaded to the same level of their rated capacity throughout their service life. This will have direct impact on their preventive and major maintenance schedules. Also this will have effect on the fuel consumption rates since the control system of these parallel operating generators have sequence logic called load demand control. This control permits each generator to be loaded up to a certain load level at which another standby generator will be called to start and synchronize with the running generators. On the opposite side, if the load on each generator is less than the minimum set point then load demand controls will shutdown the least duty generator, by which the load level on the remaining generators will be increased. The load demand and load sharing control systems ensure the parallel generators are always loaded with equal percentages of their ratings, with enough load level, giving the most economical fuel consumption rates.

The load sharing is achieved by controlling active power (P_e) output via the speed governor, and controlling reactive power (Q_e) output via the excitation controls or voltage regulator.

4.1 Load Sharing Principle:

Fig 4.1 shows the equivalent circuit of two generators operating in parallel having internal voltages of E_1, E_2 and reactances x_1 and x_2 respectively feeding a total current I_L. It can be shown that,

$$I_1 = (E_1-E_2)/(x_1+x_2) + I_L x_2/(x_1+x_2) \tag{4.1}$$

$$I_2 = (E_2-E_1)/(x_1+x_2) + I_L x_1/(x_1+x_2) \tag{4.2}$$

Where I_1 and I_2 are the currents applied by individual machines. The first term of the two equations represents the current circulating from the first machine stator to the second and vice versa. The second term represents the share of the total load current by each machine.

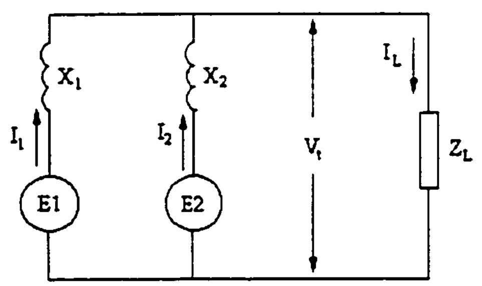

Fig 4.1 Two alternators in parallel supplying local load

Without any control, the load will be distributed according to their reactances and internal voltages as given in equations (4.1) and (4.2). When different generators are paralleled with each other, the criteria of sharing the load equally is to equal them with relative to their rated power. Droop and isochronous load sharing methods are widely used for active power sharing. In the droop mode each generator's frequency will droop by a certain percentage as the load is increased on. The maximum droop is usually attained at the full load. While in the isochronous mode the the load sharing will control the power level on each generator to achieve the load sharing leaving each generator governor to correct the frequency as the load is changed on the generator. In this case all generators will have fixed frequency.

Let us define P_i as generator # i active power in KW, P_{i_B} as generator # i rated capacity or base power in KW, and P_{e_i} as generator # i load percentage with relative to its base power (rated power). It can be shown that for active power sharing between 'n' number of parallel generators P_{e_i} of all generators must be equal or,

$$P_{e_1} = P_{e_2} = \ldots\ldots = P_{e_n} \qquad \text{in p.u} \qquad (4.3)$$

Or

$$P_i/P_{i_B} = P_{i+1}/P_{i+1_B} = \ldots\ldots\ldots = P_n/P_{n_B} \qquad \text{in p.u} \qquad (4.4)$$

Where,

$$P_{e_i} = P_i/P_{i_B} \qquad \text{in p.u} \qquad (4.5)$$

For load sharing control the instantaneous load sharing error $\Delta P_{ei_{LS}}$ is to be determined for each generator. This is defined as the diffrence between the load percent on each generator P_{e_i} and the load sharing power P_{LS} in percentage. First we write P_{LS} as,

$$P_{LS} = \Sigma_{i=1} P_i / \Sigma_{i=1} P_{i_B} \qquad \text{in p.u} \qquad (4.6)$$

Then the load sharing error is,

$$\Delta P_{ei_{LS}} = P_{LS} - P_{e_i} \qquad \text{in p.u} \qquad (4.7)$$

Similarly for reactive power sharing of 'n' parallel generators, it can be shown that,

$$Q_i/Q_{i_B} = Q_{i+1}/Q_{i+1_B} = \ldots\ldots\ldots\ldots = Q_n/Q_{N_B} \qquad (4.8)$$

Where Q_i is the instantaneous reactive power of generator # i in KVAr and Q_{i_B} is the base (or rated) power of generator # i (usually $P_{i_B} = Q_{i_B}$). The reactive power sharing is Q_{LS}

$$Q_{LS} = \Sigma_{i=1} Q_i / \Sigma_{i=1} Q_{i_B} \qquad \text{in p.u} \tag{4.9}$$

The instantaneous load sharing error for each generator $\Delta Q_{ei_{LS}}$

$$\Delta Q_{ei_{LS}} = Q_{LS} - Q_{e_i} \tag{4.10}$$

The following example will clarify the above concept. Two generators will be considered in the example and in the rest of the simulation study in this chapter.

<u>Example:</u>

Two diesel generators in parallel with the following operational data:

$P_{1_B} = 2000$ KW $\qquad P_{2_B} = 1600$ KW

Their initial active power load levels are:

$P_1 = 1200$ KW $\qquad P_2 = 1300$KW

What will be their loads after switching ON the active load sharing control ?

<u>Solution:</u>

$P_{e_1} = 1200/2000 = 0.6$

$P_{e_2} = 1300/1600 = 0.8125$

$n = 2$

$P_{LS} = (P_1+P_2)/(P_{1_B}+P_{2_B}) \quad = (1200+1300)/(2000+1600) = 0.69444$

$\Delta P_{e1_{LS}} = P_{LS} - P_{e_1} \quad = 0.6944 - 0.6 \quad = 0.09444$

$\Delta P_{e2_{LS}} = P_{LS} - P_{e_2} \quad = 0.6944 - 0.8125 \quad = -0.11805$

Then each generator power percentage P_{e_i} will be adjusted accoding to the load sharing error $\Delta P_{e1_{LS}}$ obtained for it as,

$P_{e_1}' = P_{e_1} + \Delta P_{e1_{LS}} = 0.6 + 0.09444 \qquad = 0.69444$

$P_{e_2}' = P_{e_2} + \Delta P_{e2_{LS}} = 0.8125 + (-0.11805) = 0.69444$

In KW the two generators actual load is

$P_1' = P_{e_1}' \, P_{1_B} = 0.69444 * 2000 = 1388.9\text{KW}$

$P_2' = P_{e_2}' \, P_{2_B} = 0.69444 * 1600 = 1111.1 \text{ KW}$

Now both generators are sharing the total load with 69.4%. To allocate the active load power to achieve load sharing, the integral negative feedback of the load sharing error $\Delta P_{ei_{LS}}$ of each generator should be employed in the speed control. The integral feedback is basically a speed error signal, which will be added to the proportional speed error signal. These speed error signals will tend to adjsut the output rotor mechanical power by $\Delta P_{m_{LS}}$ to compensate the amount of load sharing error $\Delta P_{ei_{LS}}$.

A practical test was implemented on two diesel generators operating in parallel and using standard speed and excitation controls. Both generators were equipped with load sharing controls. The two generators were identical and rated 1500 KW each. The common load was applied on the two generators as listed in Table 4.1. Initially, at the beginning of the test at 13:45 Hours PM the generators were sharing the load equally. The generator control settings were not touched throughout the test, where the load itself was changing as shown in Fig 4.2. The load on each generator was recorded. It was noticed that at each change of the load, the share on each generator tends to change and the load sharing error increases as shown on Fig 4.3. This is happening with the load sharing controls fitted and calibrated. We would expect what happens if the load sharing controls were not used. This experiment was carried out at Saudi Diesel Generators Company premises in Al-Khobar, Saudi Arabia just to show the seriousness of this problem even with the load sharing controls fitted, and over such relatively short period of time. The error would accumulate if these generators were left operating continuously for several days. From the experience it is found that the automatic load sharing has to be adjusted regularly on the generators.

The load sharing error comes up due to the differences in the dynamics of the two generators, the difference in their impedance, the accuracy of the sensors, the calibration set points drifting. Although these generators were designed to be identical, the load sharing error in this case was very obvious.

TIME	GEN-1 KW	GEN-2 KW	TOTAL LOAD KW	P_{LS}	ERROR G-1 %	ERROR G-2 %	f_1 HZ	f_2 HZ
1345	250	250	500	250	0.00%	0.00%	60	60.03
1350	247	253	500	250	1.20%	-1.20%	60.03	60.03
1355	516	484	1000	500	-3.20%	3.20%	60.05	60.05
1400	514	486	1000	500	-2.80%	2.80%	60.03	60.05
1405	758	742	1500	750	-1.07%	1.07%	60.07	60.07
1410	784	716	1500	750	-4.53%	4.53%	60.05	60.07
1415	786	714	1500	750	-4.80%	4.80%	60.05	60.07
1420	1265	1135	2400	1200	-5.42%	5.42%	60.07	60.09
1425	1267	1133	2400	1200	-5.58%	5.58%	60.07	60.07
1430	1268	1132	2400	1200	-5.67%	5.67%	60.07	60.09
1435	1268	1132	2400	1200	-5.67%	5.67%	60.07	60.09
1440	1264	1136	2400	1200	-5.33%	5.33%	60.09	60.09
1445	1263	1137	2400	1200	-5.25%	5.25%	60.07	60.1
1450	1259	1141	2400	1200	-4.92%	4.92%	60.09	60.09
1455	1262	1138	2400	1200	-5.17%	5.17%	60.09	60.11
1500	757	743	1500	750	-0.93%	0.93%	60.07	60.07
1505	758	742	1500	750	-1.07%	1.07%	60.07	60.07
1510	760	740	1500	750	-1.33%	1.33%	60.07	60.07
1515	758	742	1500	750	-1.07%	1.07%	60.07	60.07
1520	757	743	1500	750	-0.93%	0.93%	60.07	60.07
1525	761	739	1500	750	-1.47%	1.47%	60.07	60.05
1530	761	739	1500	750	-1.47%	1.47%	60.07	60.07
1535	760	740	1500	750	-1.33%	1.33%	60.07	60.09
1540	759	741	1500	750	-1.20%	1.20%	60.07	60.07
1545	758	742	1500	750	-1.07%	1.07%	60.07	60.07
1550	761	739	1500	750	-1.47%	1.47%	60.07	60.07
1555	761	739	1500	750	-1.47%	1.47%	60.07	60.07
1600	760	740	1500	750	-1.33%	1.33%	60.07	60.07

Table 4.1 The two SDGC generators load sharing test record

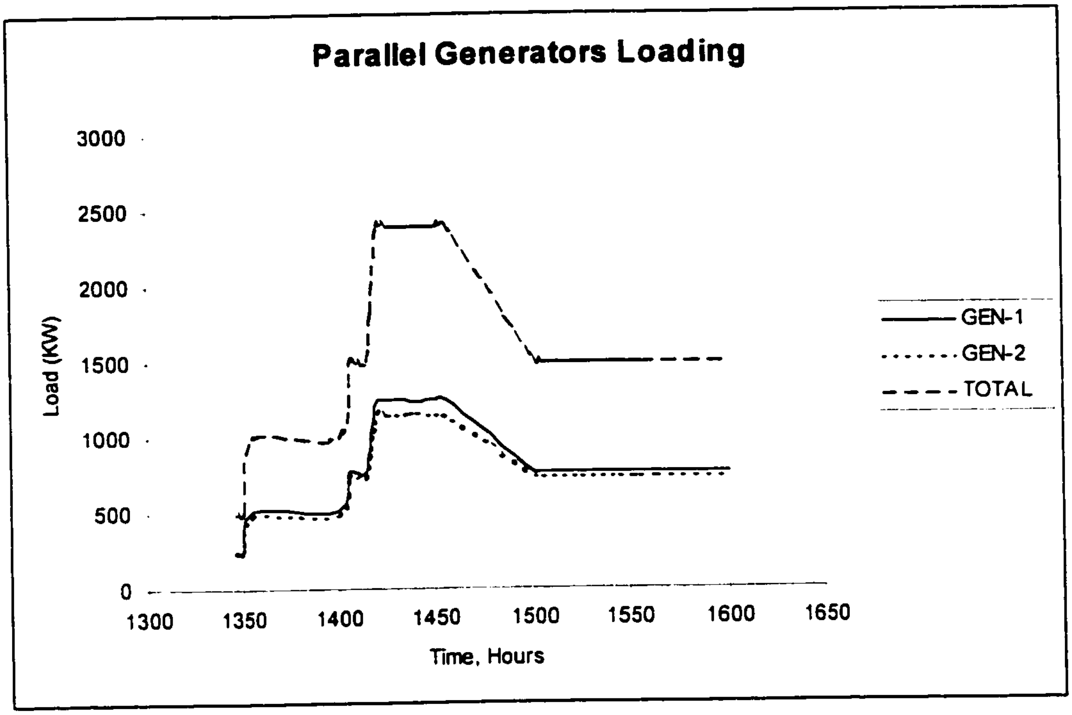

Fig 4.2 Generator # 1 and 2 load change with time

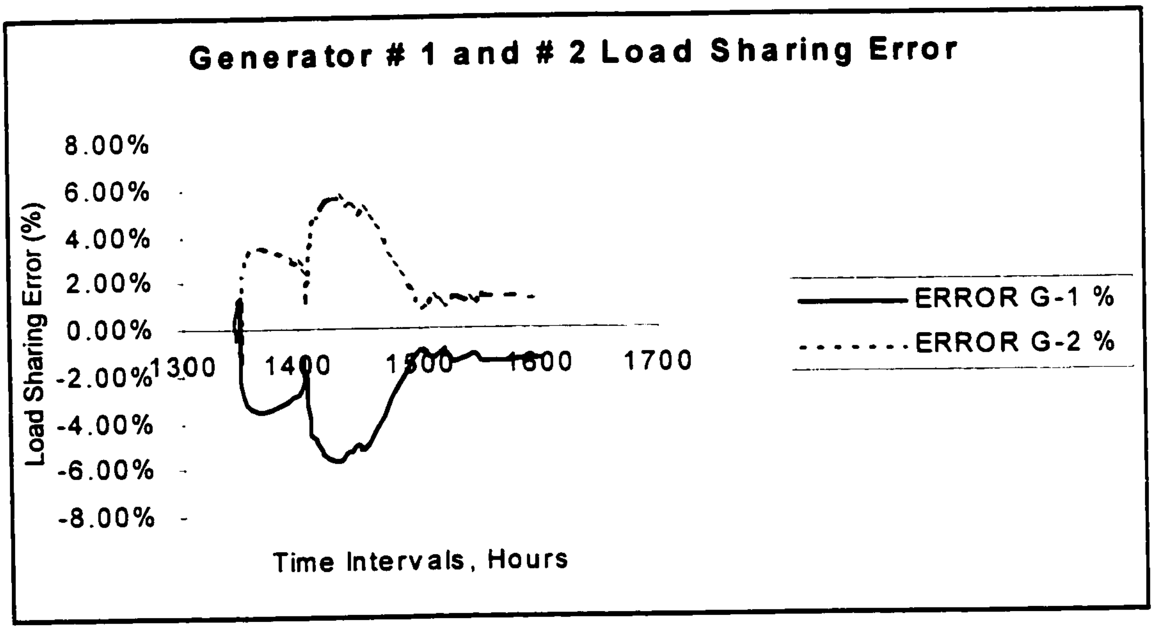

Fig 4.3 Generator # 1 and # 2 load sharing error %

4.2 Droop and Isochronous Load Sharing:

In older paralleling systems, droop characteristics shown in Fig 4.4 was employed to achieve stability (eliminate hunting) and proportion load division. In such systems, the actual line voltage and frequency is a function of the per cent of the full load connected to the bus. Thus, generators are added to the bus under close supervision to achieve proper bus stability, minimize synchronizing transients, and balance loading on individual sets. Such arrangements are not particularly suited to dynamic load fluctuations of any appreciable size, or to fully automatic unattended operation.

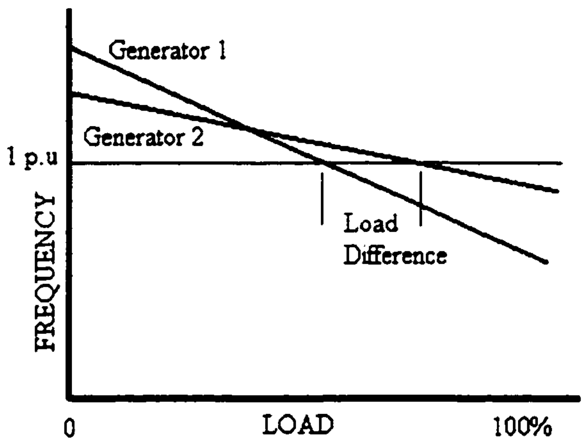

Fig 4.4 Speed Droop Operation

However, today's diesel engine generator sets can be furnished with relatively simple and reliable electronic governors and voltage regulators, which make these sets particularly suitable for unattended automatic paralleling and load sharing. The electronic governor provides isochronous operation characteristics as shown in Fig 4.5. This provides constant speed regardless of load, and automatic proportionate load division, which enable automatic paralleling of dissimilar size sets.

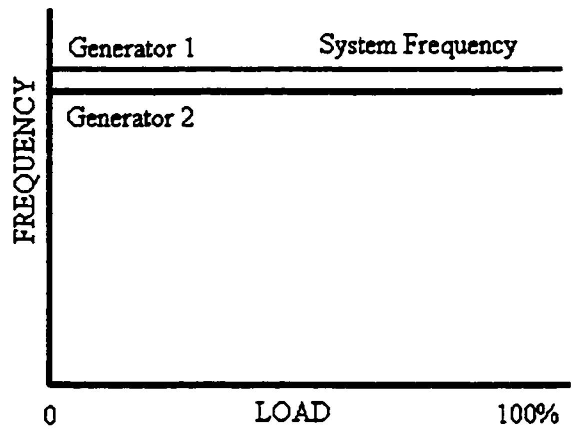

Fig 4.5 Speed Isochronous Operation

4.3 Load Sharing Control System:

As stated in section 4.1 the active power sharing is controlled through speed governor, while the reactive power sharing is controlled through excitation and voltage regulator. In the following sections the two control schemes are explained and modeled.

4.3.1 Active Power Sharing Control:

In general, the load sharing control involves two steps, the first step is to measure the active power and calculate the load sharing error $\Delta P_{ei_{LS}}$ for each generator. This is obtained by subtracting each generator load from the load sharing power P_{LS} of all parallel generators as defined in equation (4.7). The second step of the load sharing control is to convert $\Delta P_{ei_{LS}}$ to rotor speed error $\Delta\omega_{i_{LS}}$, by integrating $\Delta P_{ei_{LS}}$ to get the $\Delta\omega_{i_{LS}}$.

$\Delta\omega_{i_{LS}}$ is fed back negative to the speed controller input as shown in Fig 4.6

The basic control law consists of two parts, the negative proportional feedback control of the speed error $\Delta\omega_i$ and the negative integral feedback control of the load share power error ΔP_{LS_i}. This is written as [16]:

$$\Delta\omega_T = K_{gp}\Delta\omega_i - (1/s)K_{gI}\,\Delta P_{LS_i} \qquad (4.11)$$

where K_{g_p} is the proportional feedback gain of the speed error, and K_{g_I} is the intergral feedback gain of the active power error. Ideally K_{g_I} should be set to equal 1/M. For the purpose of simulation we set t to 1/M. In this case the load sharing loop will be the same as the swing equation loop, where ΔP_{LS} can be added to P_e to simulate the load sharing effect.

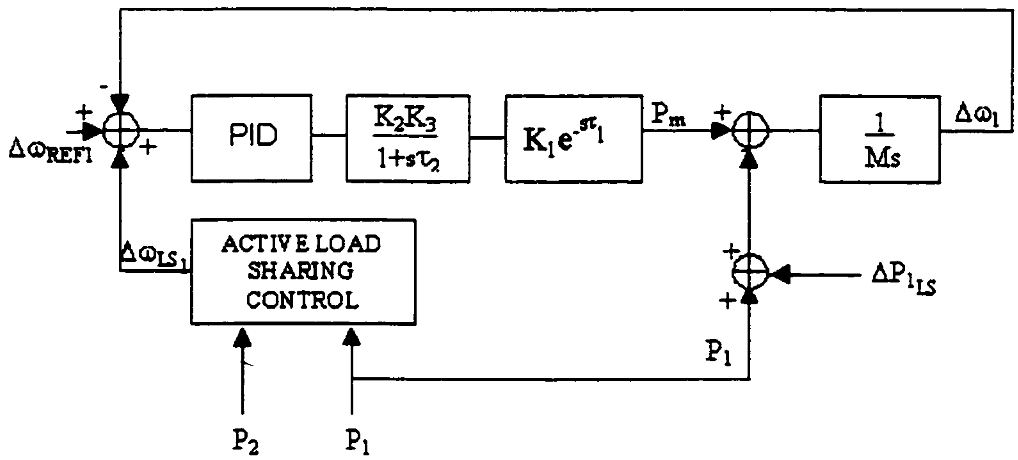

Fig 4.6 KW load sharing ontrol for generator # 1

4.3.2 Reactive Load Sharing Control:

Similar to the real load sharing, the reactive load-sharing error $\Delta_{Qei_{LS}}$ is integrated to get the voltage setting error $\Delta V_{i_{LS}}$, which is fed back negative into the excitation control as shown in Fig 4.7.

The basic control law in this case consists involves two parts, the negative proportional feedback control of the voltage error ΔV_i and the negative integral feedback control of the load share power error ΔQ_{LS_i}. This is written as [16]:

$$\Delta V_T = K_{e_p}\Delta V_i - (1/s)K_{e_I}\,\Delta Q_{LS_i} \tag{4.12}$$

where K_{ep} is the proportional feedback gain of the terminal voltage error, and K_{e_I} is the intergral feedback gain of the reactive power error.

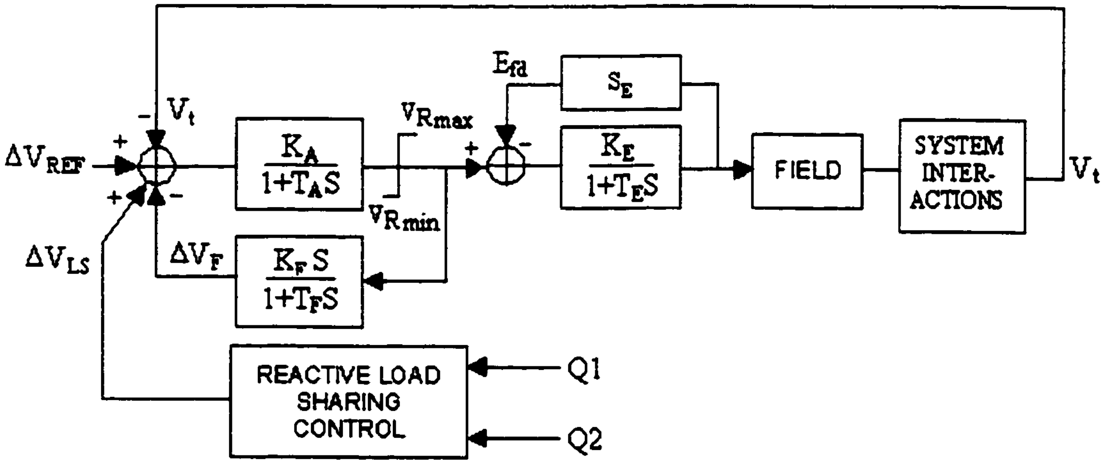

Fig 4.7 KVAr Load sharing control

4.3.3 How Load Sharing Controls function:

When two generators are in parallel and share active load unequally, then as soon as the load sharing control is switched ON, each generator load sharing control will detect its ΔP_{eiLS}. The generator which shares less than what is supposed to be will tend to increase its output power by increasing its mechanical torque. This will allow the generator to carry some of the available electrical load on the common bus-bar, such that the speed of the generator is maintained constant (isochronous operation). In reality this is achieved with some oscillation until the speed stabilizes. The speed control does not adjust the electrical power directly, but does so indirectly by increasing the mechanical power and accelerating the rotor, which opens a window for the electrical power to incresae or decrease on the generator until the acceleration power becomes zero. Similarly in reactive power load sharing, ΔQ_{eiLS} is detected. This will tend to increase or decrease each generator excitation voltage to compensate for the error. This activates to adjust the terminal voltage.

4.4 Simulation of Load Sharing Controls:

Two identical generators are operating in parallel, these have the ratings and other parameters as DG-1 detailed in Appendix-A. The generators are initially loaded unequally as follows:

P_1 = 600 KW $\quad$ Q_1 = 53 KVAr $\quad$ P_{1_B} = 1375 KVA

P_2 = 1000 KW $\quad$ Q_2 = 1147 KVAr $\quad$ P_{2_B} = 1375 KVA

The initial load for generator # 1 is 0.44 p.u, while for generator # 2 the initial load is 0.73 p.u. For them to share the load equally, they must be loaded up to 0.58 which is 800 KW on each generator (Base power = 1375). Similarly generator # 1 initial reactive load is 0.04 p.u, while for generator # 2 it is 0.83 p.u. For equal KVAr sharing each generator must be loaded up to 0.436. This will give 600 KVAr for each generator (Base power = 1375).

The load sharing control was switched ON at time = 2 second. At time = 4 seconds the system is in steady condition when the active and reactive powers were redistributed equally on the two generators. The new loading conditions become:

P_{e_1} = 800 KW $\quad\quad$ Q_{e_1} = 600 KVAr

P_{e_2} = 800 KW $\quad\quad$ Q_{e_2} = 600 KVAr

The simulation was carried out on Matlab. The algorithm is detailed in Appendix-D.5. The results are plotted in Fig 4.8, 4.9, 4.10 and 4.11.

Fig 4.8 shows the response of the two generators frewuency deviations as affected by the load sharing control error correcting action.

Fig 4.9 shows how the two generators mechanical powers are also affected to match the load sharing control correcting actions.

Fig 4.10 shows to the two generators electrical power output in KW and how they are corrected by the load sharing control.

Fig 4.11 shows the two generators reactive power output in KVAr and how they are corrected by the load sharing control.

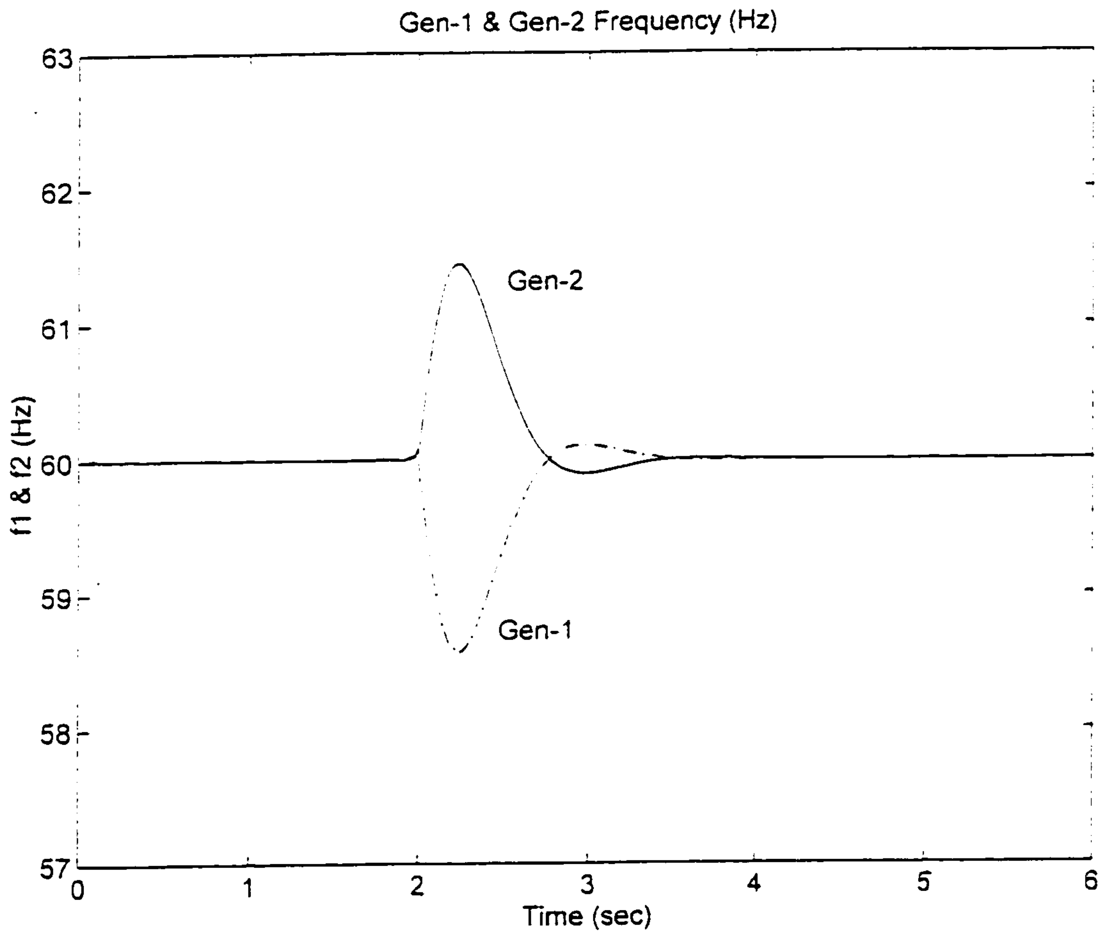

Fig 4.8 Two identical parallel generators with load sharing control
Switched ON at time = 2 seconds.
Generator # 1 and # 2 frequency deviation response
(- - - Gen-1 —— Gen-2)

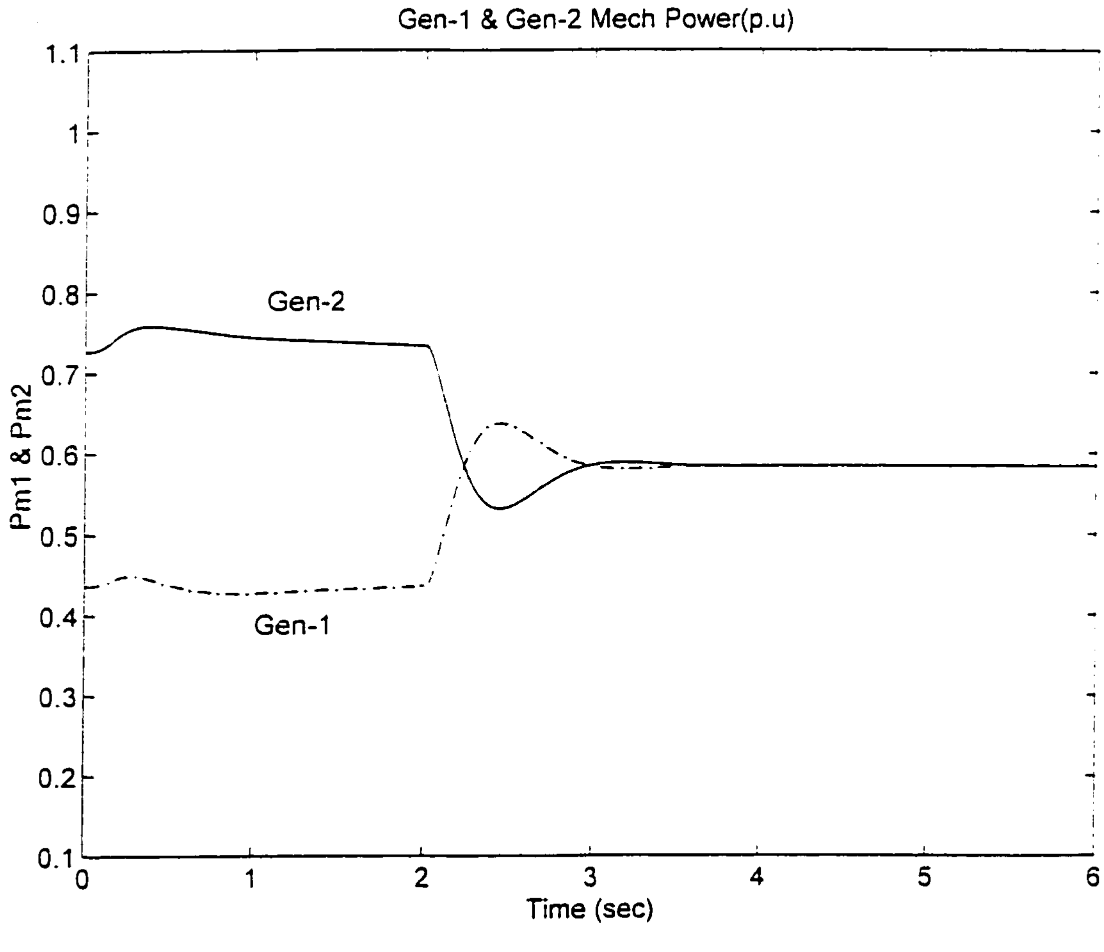

Fig 4.9 Two identical parallel generators with load sharing control
Switched ON at t=2 seconds.
Generator # 1 and # 2 mechanical power response
(- - - Gen-1 —— Gen-2)

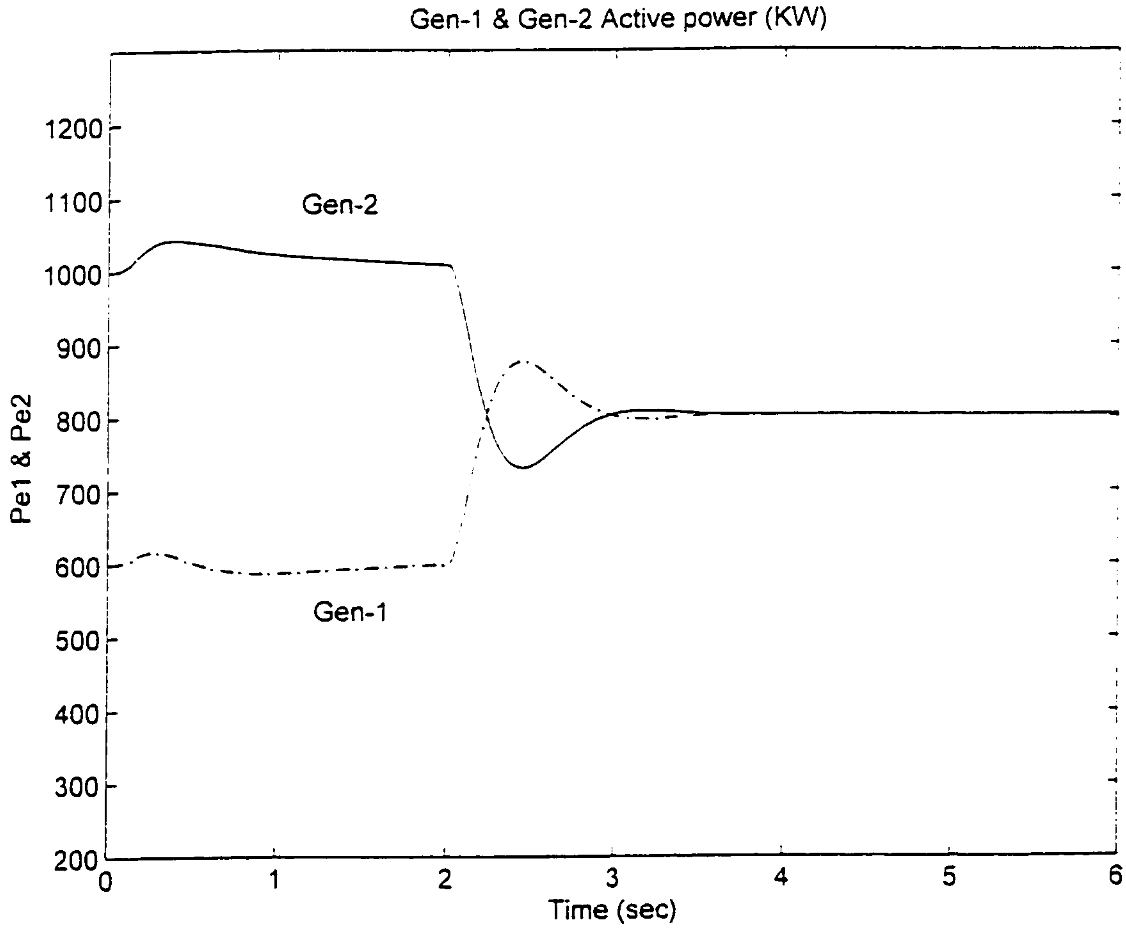

Fig 4.10 Two identical parallel generators with load sharing control Switched ON at t=2 seconds.
Generator # 1 and # 2 active power response in KW
(- - - Gen-1 —— Gen-2)

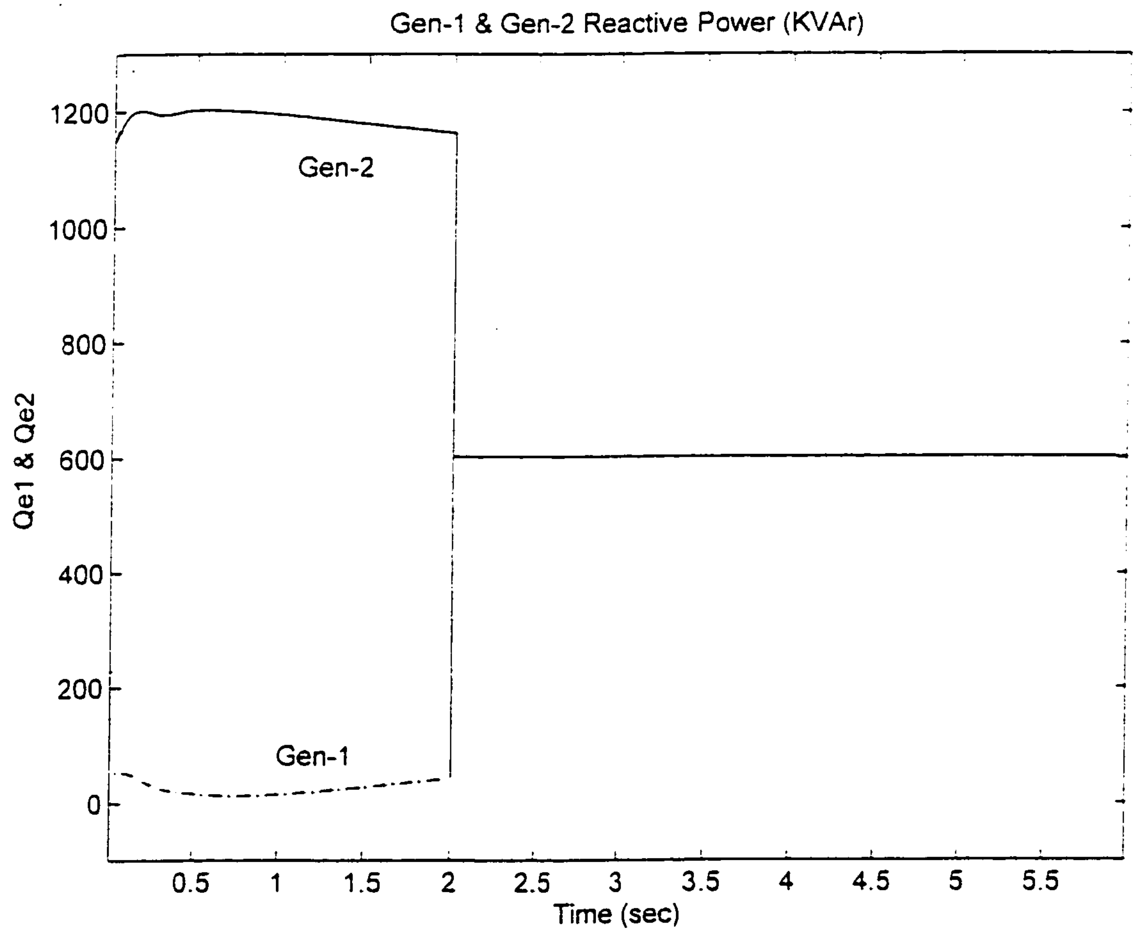

Fig 4.11 Two identical parallel generators with load sharing control Switched ON at t=2 seconds.
Generator # 1 and # 2 reactive power response in KVAr
(- - - Gen-1 —— Gen-2)

Chapter 5

CENTRALIZED CONTROLLER USING LQR METHOD WITH SYSTEM'S SINGULAR VALUES-BASED WEIGHTS

The LQR design problem has been extensively investigated for the past four decades. Consequently, the theory of linear optimal control has been very well developed. Central to the optimal stabilizer design problem are the formulation of the cost function and the choice of the state and control weighting matrices. Although the need for the design of a cost function that reflects the physical characteristics and practicalities of the studied system is reasonably understood, the choice of the elements of the weighting matrices that would translate this understanding still remains a difficulty. This is mainly due to the interactive nature of the states and controls of linear multivariable dynamic systems like in power system [17]. The available methods are all based on trial and error and do not offer a systematic way of choosing Q and R. The best approaches so far depend a lot on the designer's experience.

The LQR design consists of finding a feedback control law minimizing the cost function $J=\int(x^T Qx+u^T Ru)dt$, where $Q \geq 0$, $R > 0$, subject to the state space equation $\dot{x} = Ax + Bu$, $y = Cx$. Where u is the r-dimension control vector , and y is the p-dimension output vector.

5.1 The New LQR Approach

A new Approach that calculates the Q and R matrices from the system state equations has been recently proposed. The algorithm as detailed by M. Aldeena and F. Crusca [17] is to transform the system into an ordered balanced form for which the controllability and observability Gramians are equal and diagonal. The usefulness of transforming the system into the balanced form is that its elements will be ordered according to their combined measure of controllability and observability, reflected by its associated Hankel Singular Values (HSV's). The most controllable and observable state will appear as the first element in the state vector of the balanced system. The least controllable and observable state will appear as the last element in the state vector.

Consider the state space transformation

$$x_b(t) = \mathbf{T}\, x(t) \tag{5.1}$$

where $x_b(t)$ is the state vector of the balanced form and **T** is the balanced transformation matrix. Then we can write the dynamic state equations as follows:

$$\dot{x}_b(t) = A_b\, x_b(t) + B_b\, u(t) \tag{5.2a}$$

$$y_b(t) = C_b\, x_b(t) \tag{5.2b}$$

where

$$A_b = TAT^{-1} \qquad B_b = TB \qquad C_b = CT^{-1} \tag{5.2c}$$

Then the controllability and observability Gramians W_C and W_O must be equal and diagonal, where the diagonal values are called the HSV's as follows:

$$W_c = W_o = \mathrm{diag}\,(\sigma_1, \sigma_2, \ldots, \sigma_m, \sigma_{m+1}, \ldots\ldots, \sigma_n), \quad \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \tag{5.3}$$

σ_i's are the positive square root of the eigenvalues of W_oW_c

This transformation will be used to find Q and R matrices.

5.1.1 Q-Matrix:

Based on the above, the Q matrix is calculated as follows

First: The first m states of the balanced system are those states, which are deemed to contribute most to the dynamical behavior of the system and they should be weighted according to their contribution.

Second: The n-m states of the balanced system shall be ignored, by placing zero weighting on them, because they are weakly controllable and observable. Due to the fact that say the j^{th} state requires less control effort than the following states, it is wise that each state should be weighted according to the ratio of its contribution with respect to the most controllable and observable one. Therefore Q_b is obtained first then transformed back to get Q as follows:

$$Q_b = \mathrm{diag}\,(1, \sigma_1/\sigma_2, \sigma_1/\sigma_3, \ldots, \sigma_1/\sigma_n) \tag{5.4}$$

$$Q = T^* Q_b T \tag{5.5}$$

Where T^* is the conjugate transpose of T.

5.1.2 R-Matrix:

In section 5.1.1 the method of obtaining the state matrix Q was based on proper weighting of the states of the balanced form system. In this section the control matrix R will be similarly obtained from the weighting of the control inputs. This is obtained as follows:

1. The original system shall be partitioned in terms of its control inputs as follows:

$$\dot{x}(t) = A\,x(t) + \sum b_i\,u_i(t) \qquad i = 1,\ldots r \tag{5.6}$$

$$y(t) = I_n\,x(t) \tag{5.7}$$

where I_n is the identity matrix of order n

2. Transform each i^{th} single input multi-output system into its respective balanced form.

3. Obtain the controllability and observability gramians corresponding to each i^{th} control input. These are W_{C_i} and W_{O_i} which are equal.

4. Calculate the contribution of the i^{th} control input ω_i which is the summation of the diagonal entries of the corresponding W_{Ci} or W_{O_i}

5. Calculate γ which is a positive scalar constant determining the tightness of the control action. This is set to:

$$\gamma = \omega_1/\sigma_1 \tag{5.8}$$

6. Form the matrix R as:

$$R = \gamma\,(1, \omega_2/\omega_1, \omega_3/\omega_1, \ldots\ldots, \omega_r/\omega_1) \tag{5.9}$$

5.2 Centralized LQR Controller Algorithm:

In this sction a centralized LQR controller will be developed for two parallel generators DG-1 and DG-2. The first step is to simulate both generators as stand-alone generators using the model developed in Chapter 2, with and without LQR control for comparison purpose from which a de-centralized LQR controller is developed for the two generators. Then the two parallel generators model, which was simulated in Chapter 3 will be simulated with and without centralized LQR. Response of generators in all cases will be plotted and compared.

5.2.1 DG-1 Test:

The same algorithm, which was used in section 2.7 will be reused to simulate DG-1 as detailed in Appendix-D.6. Fig 5.1 dotted graphs show the response of frequency, terminal voltage, active and reactive power of the genereator to a step change in active power for the normal case (without LQR).

Then Q_1 and R_1 are selected as detailed in Appendix-D.6 to be used to calcualte the optimal feedback gain matrix K using *Matlab* lqr algorithm, which minimizes the cost function

$$J = \int (x^T Q x + u^T R u) dt \tag{5.10}$$

Subject to the constraint equation [23]:

$$\dot{x} = Ax + Bu \tag{5.11}$$

[K,S,E]=lqr(A1,B1,Q1,R1) returns **S**, the unique positive definite solution to the Riccatti equation [23]:

$$0 = SA_1 + A_1^T S - SB_1 R_1^{-1} B_1^T S + Q_1 \tag{5.12}$$

E the closed-loop eigen values E=eig(A1-B1*K) the optimal state feedback gain K. The new state matrix A_{1_F} becomes:

$$A_{1_F} = A_1 - B_1 * K \tag{5.13}$$

DG-1 generator is tested again with LQR control and Fig 5.1 continuous graphs give comparison between control using LQR and without LQR. This shows improvement in transient response of the generator using LQR controller.

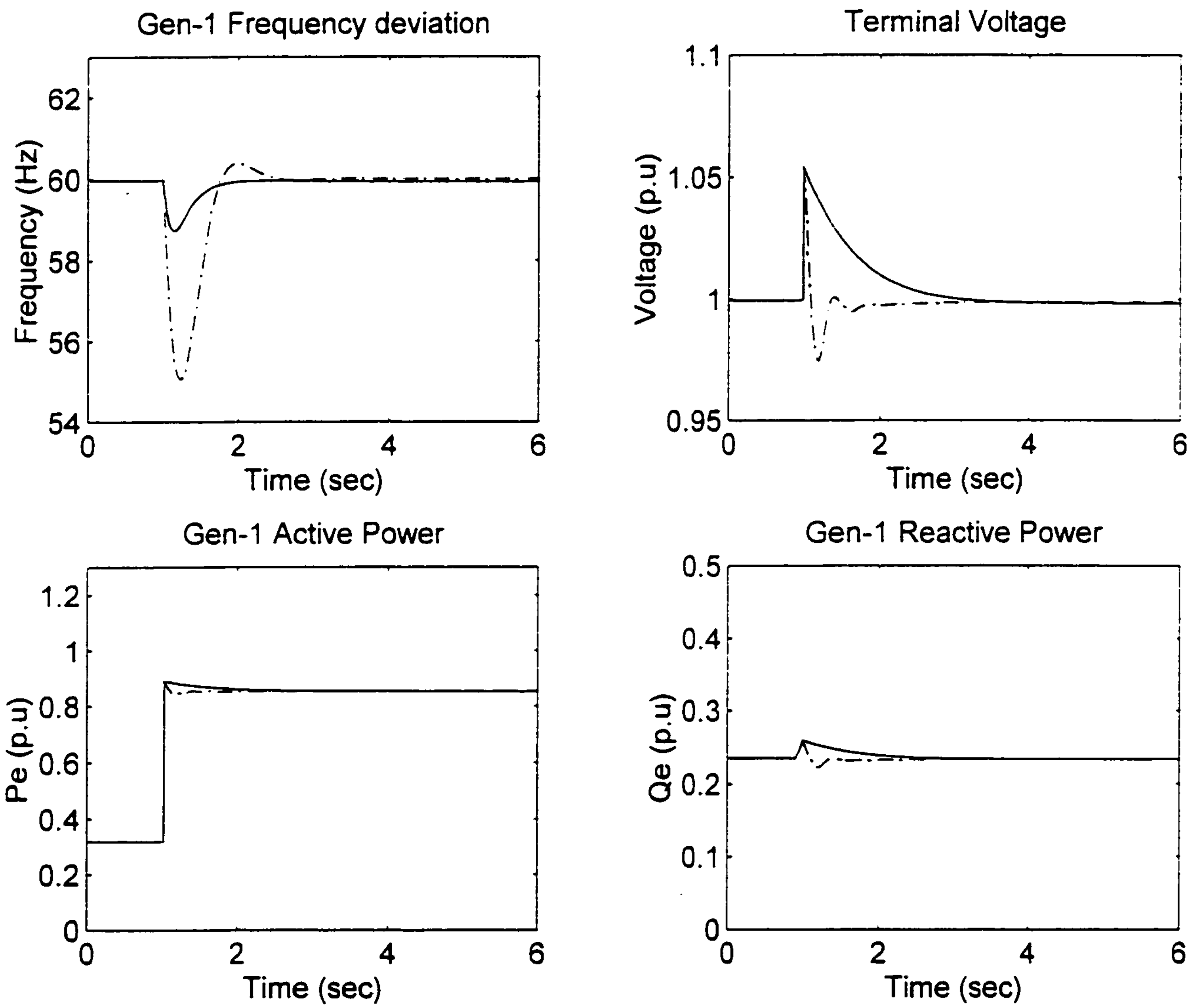

Fig 5.1 Response of DG-1 to a step change in active power

- - - Without LQR —— With LQR

5.2.2 DG-2 Test:

Similarly the above simulation of DG-1 is repeated for DG-2 and the responses are plotted in Fig 5.2. Same thing was observed as in DG-1 case.

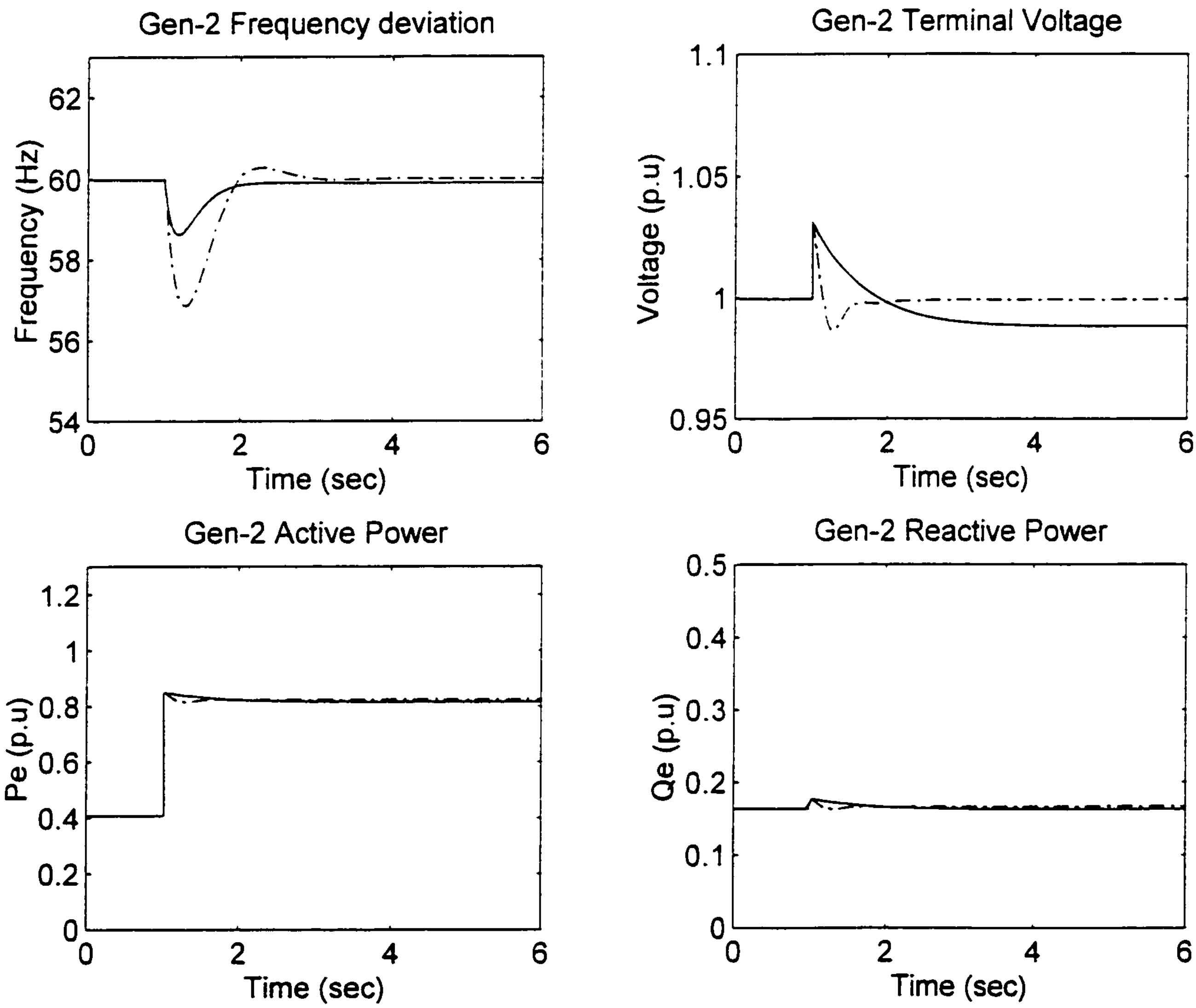

Fig 5.2 Response of DG-2 to a step change in active power

- - - Without LQR —— With LQR

5.2.3 DG-1 & DG-2 in parallel with de-centralized control:

The de-centralized LQR control is constructed by combining the two state feedback matrices obtained in the stand-alone operation cases of the two generators and form one state-feedback matrix, called the de-centralized state feedback control matrix of the two parallel generators. The model used for the two parallel generators is the linear model, which was developed in Chapter 3. See Appendix-D.6 for detailed *Matlab* algorithm. The response of the two parallel generators parameters to a step change in active load power of Fig 5.3 shows completely unstable system. This can be read from the eigen values of the de-centralized control state matrix A_{KDC} (see Appendix-D.6). The eigen values are: -4672.7, 158.8, -65.0, 21.5, -21.3, -11.2+j14.1, -11.2-j14.1, -7.4+j12.7, -7.4-j12.7, -1.0+j4.2, -1.0-j4.2, -4.9+j3.5, -4.9-j3.5, -1.5, -1.8, -2.2).

Therefore the de-centralized controller is not suitable for this system.

5.2.4 DG-1 & DG-2 in parallel with centralized LQR control:

The method used to obtain the state feedback matrix for each stand-alone generator will be reused to calculate the state feedback matrix for the two parallel generator model, which will form the centralized LQR control. This will depend on the parallel generators state matrix A_p, the inputs matrix B_p and the assigned Q_p and R_p matrices.

The procedure is detailed in Appendix-D.6 using lqr in the control tool box, where

[KP,S,E] = lqr(AP,BP,QP,RP) (5.14)

KP Centralized Control State Feedback Matrix

AKP = AP-BP*KP Centralized Control Feedback State Matrix (5.15)

The two generators system is tested again with the centralized LQR control and the response is plotted in Fig 5.4. In this figure the response is plotted with and without LQR control for comparison. There is very obvious improvement in the responses. This proves that centralized LQR controls are very suitable for the control of the parallel diesel generators system.

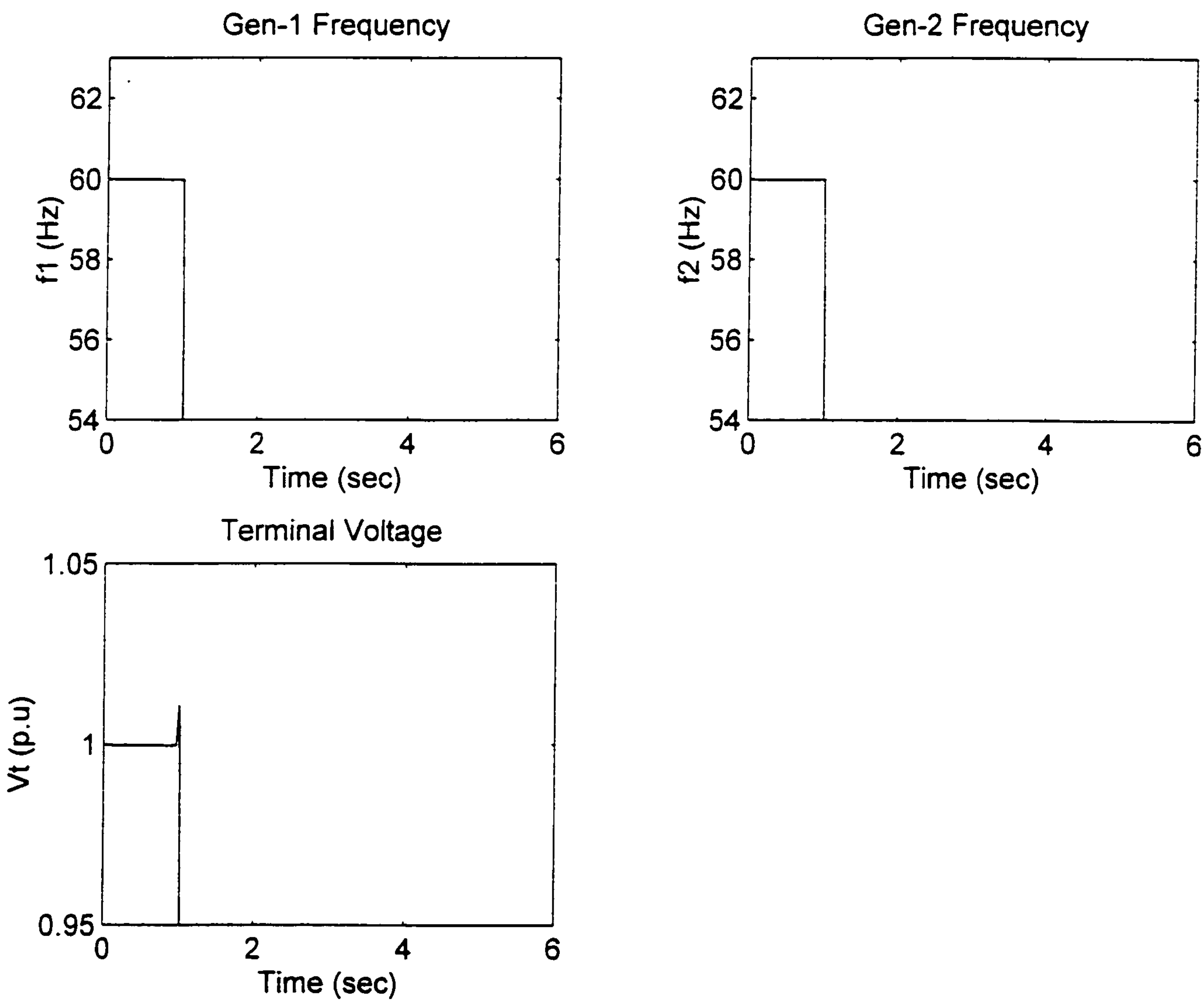

Fig 5.3 Response of parallel generators to a step change in load active power
With De-Centralized LQR Control

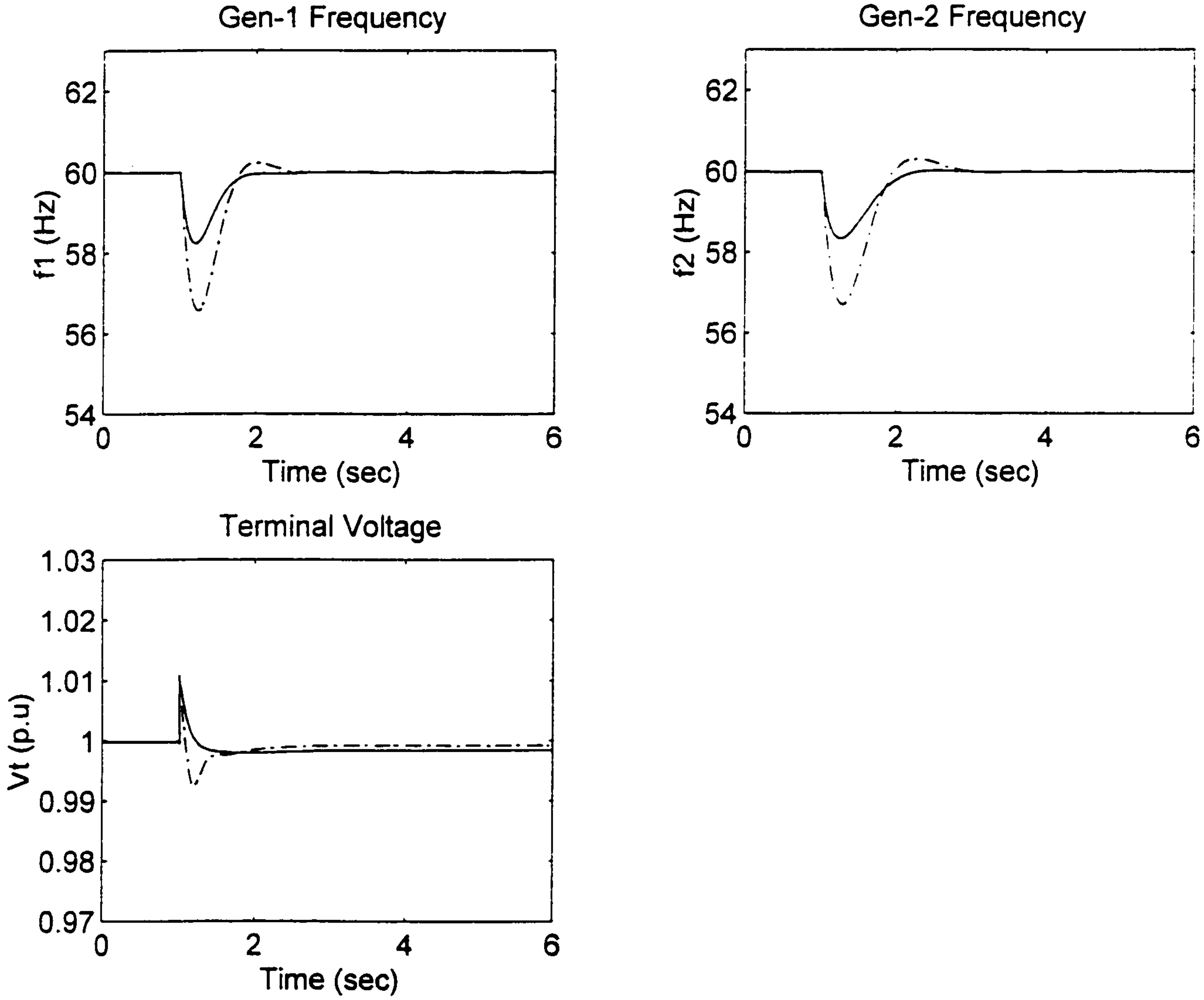

Fig 5.4 Response of parallel generators to a step change in load active power

- - - Without LQR ___ With Centralized LQR

5.2.5 LQR control with system's singular values-based weights:

The algorithm is proposed by Aldeen and Crusca [17]. The method was detailed in Section 5.1 of this chapter. The algorithm is implemented in Matlab program and detailed in Appendix-D.6. The two generators system is tested again with the calculated Q and R matrices for the LQR control. Different cases are taken depending on the singular values weights. In each case the response of the genrators will be plotted to see how effective it is. The same two generators used in the previous section tests will be used here to test this LQR method.

From Appendix-D.6 algorithm results we list all singular values σ_i's as calculated by the algorithm. These are 16 values as follows:

5.766782, 0.148856, 0.142460, 0.128267, 0.099591, 0.064132, 0.054360, 0.043183, 0.032377, 0.015443, 0.006356, 0.005996, 0.004858, 0.001847, 0.000089, 0.000006

And their weigts σ_1/σ_i are calculated by the algorithm as in equation (5.4) as follows:

$$Q_b = \begin{matrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 39 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 45 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 58 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 106 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 133 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 178 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 373 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 907 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 961 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1187 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3122 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 64498 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9193130
\end{matrix}$$

We shall consider few cases here. We shall start from the weights which correspond to the least dominant states and set it to zero and proceed until we obtain satisfactory response. We proceed as follows

Case # 1 : Q_b matrix will consider all 16 weights. The response is plotted in Fig 5.5. The response shows excellently damped system with 0.1 Hz steady state error in the generator frequency and 1% steady state error in the terminal voltage.

Case # 2 : Set σ_l/σ_{16} to zero we get the same performance as in case # 1 as shown in Fig 5.6.

Case # 3 : Set σ_l/σ_{15} and σ_l/σ_{16} to zero. As shown in Fig 5.7 this case gives worse response than in case # 1 and # 2, where frequency dips by 0.2 Hz and stay with 0.2Hz steady state error. The voltage has the same response as in the two previous cases.

Case # 4 : Set σ_l/σ_{14} , σ_l/σ_{15} and σ_l/σ_{16} to zero. Similar to case # 3 with frequency dip of 0.6 Hz. The response is shown in Fig 5.8.

Case # 5 : Set σ_l/σ_{13} , σ_l/σ_{14} , and σ_l/σ_{16} to zero ($\sigma_l/\sigma_{15} \neq 0$). This will give stable response. See Fig 5.9

In all above cases it is noticed that the terminal voltage was stable with relatively big steady satte error which equals approximately to 1%. Integrator may be required in this case to extensguish the error. It is noticed from the above that case # 2 gives the best response.

A comparative plot between case # 2 and the standard centralized LQR control is shown in Fig 5.10

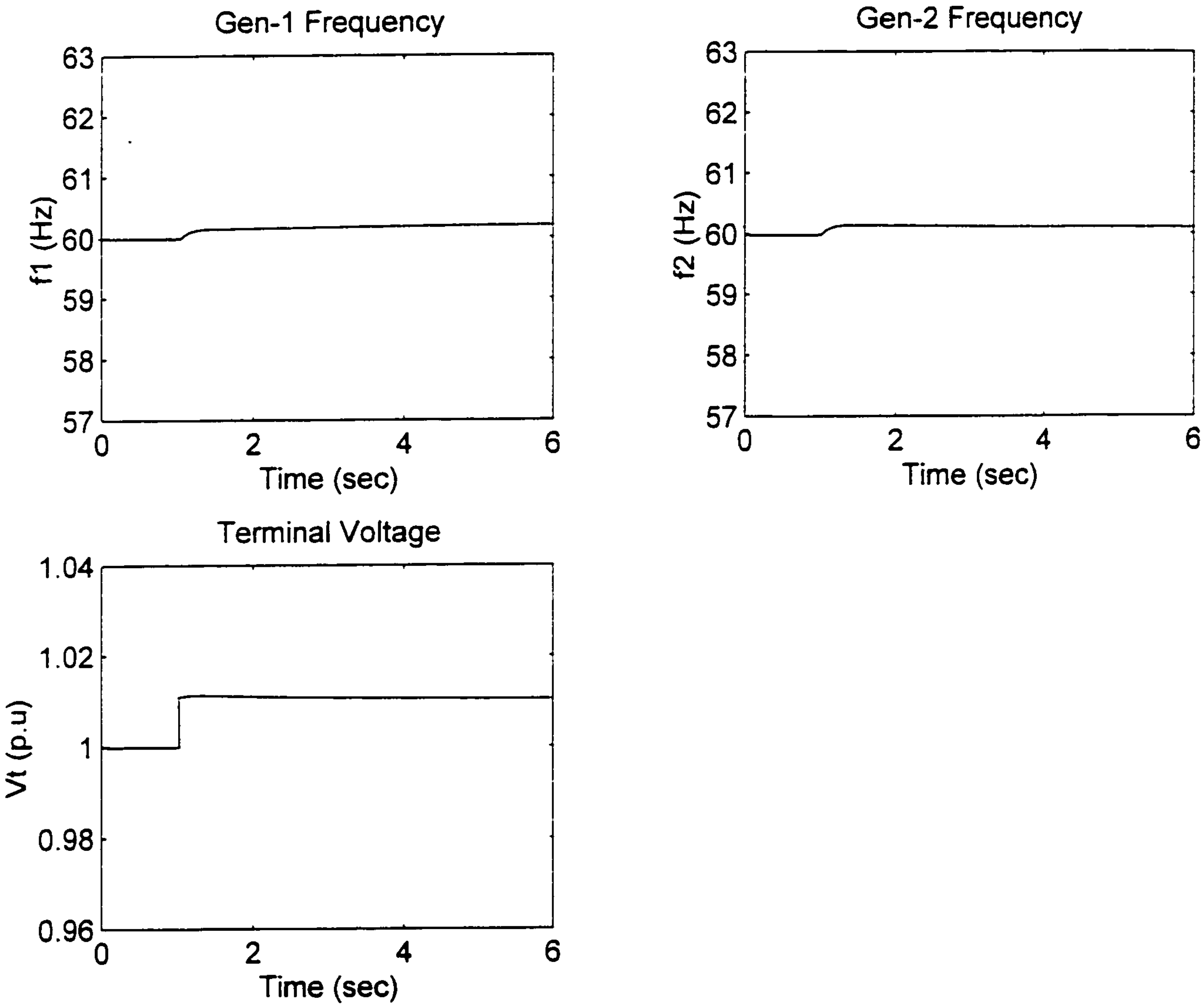

Fig 5.5 Parallel generators response to a step change in load active power with weighted Q & R matrices LQR control. Case # 1: All weights are considered

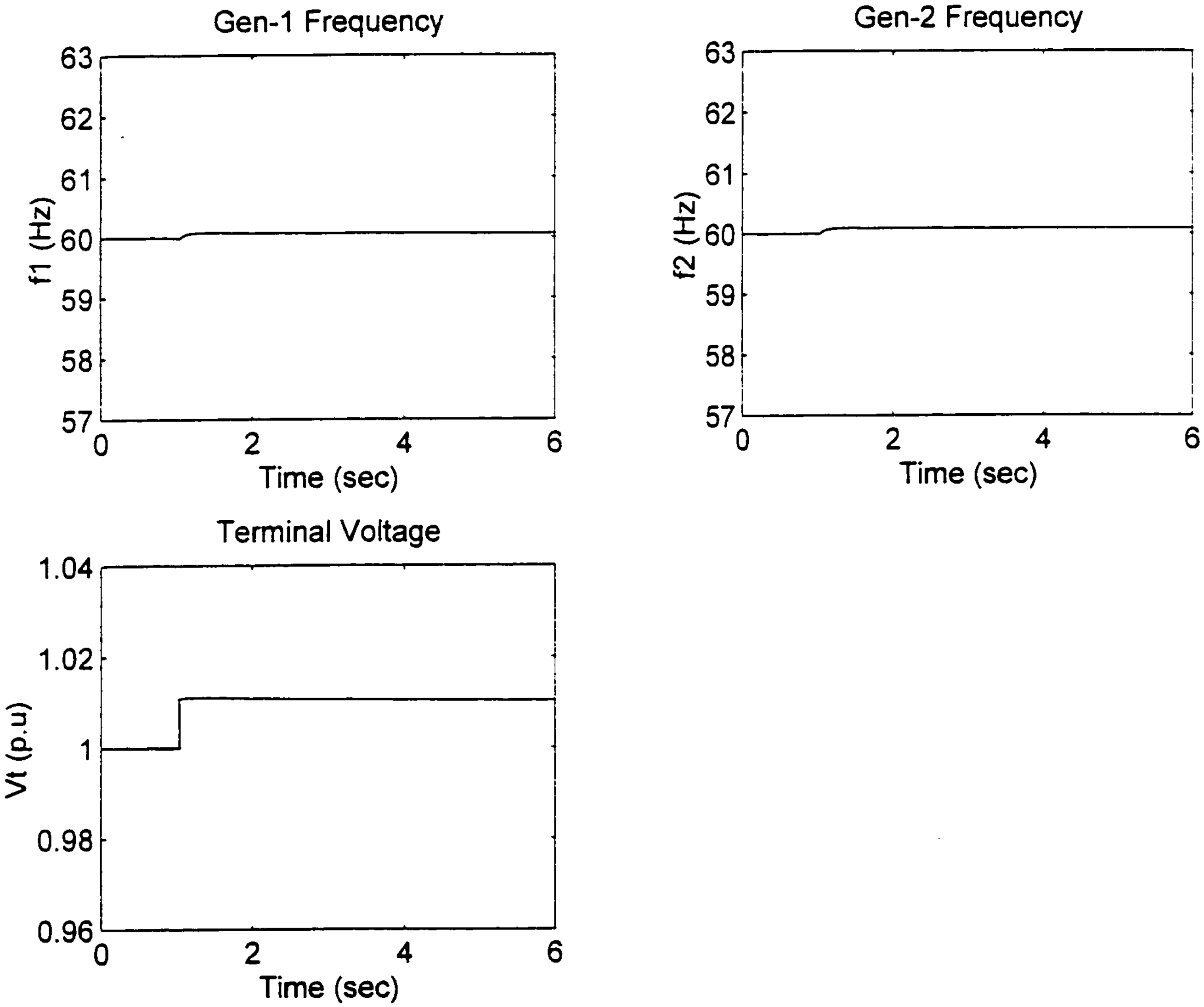

Fig 5.6 Parallel generators response to a step change in load active power, with weighted Q & R matrices LQR control. Case # 2: weight # 16 is set to zero

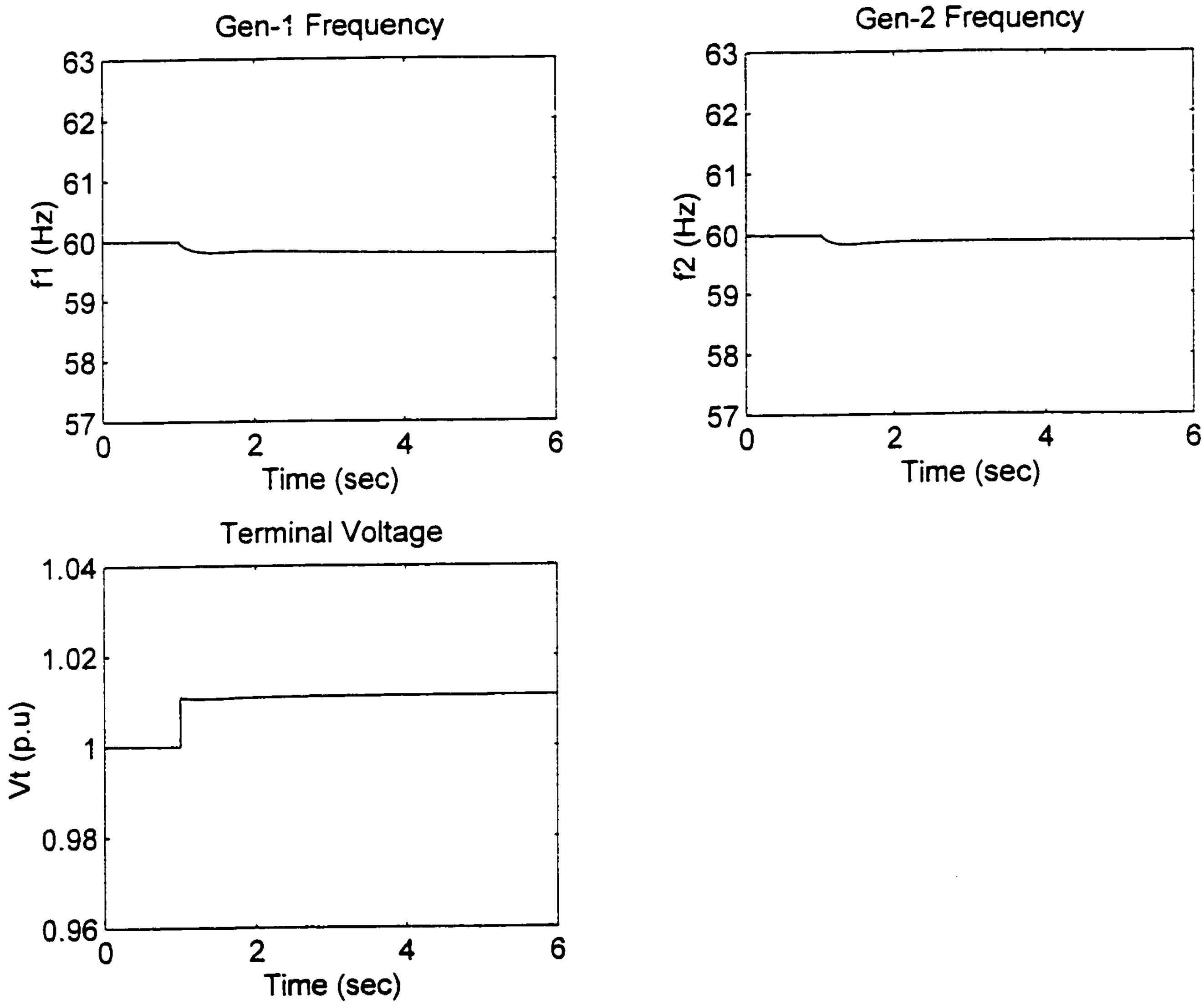

Fig 5.7 Parallel generators response to a step change in load active power, with weighted Q & R matrices LQR control. Case # 3: weight # 15 &16 are set to zero

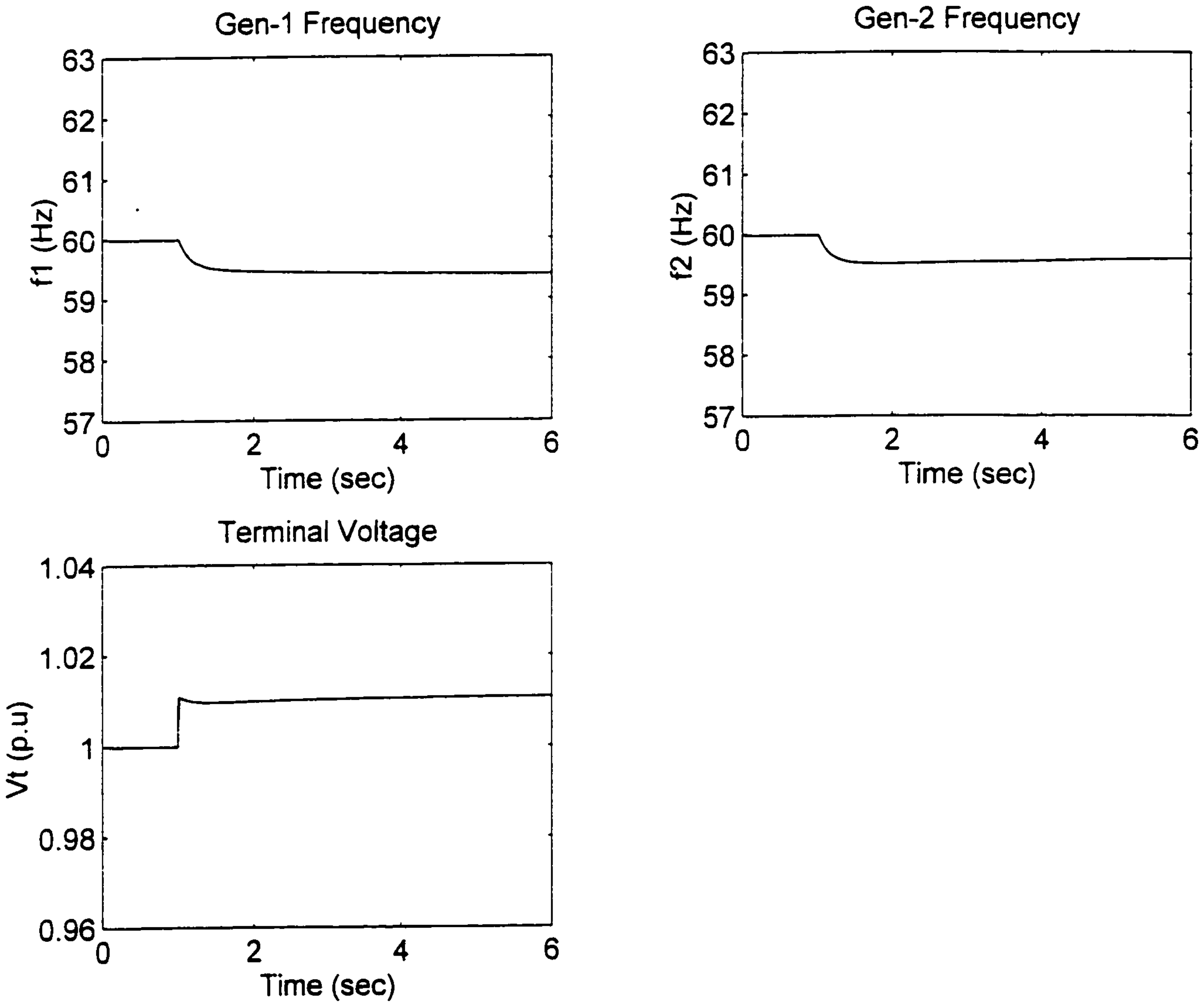

Fig 5.8 Parallel generators response to a step change in load active power, with weighted Q & R matrices LQR control. Case # 4: weight # 14, 15 & 16 are set to zero

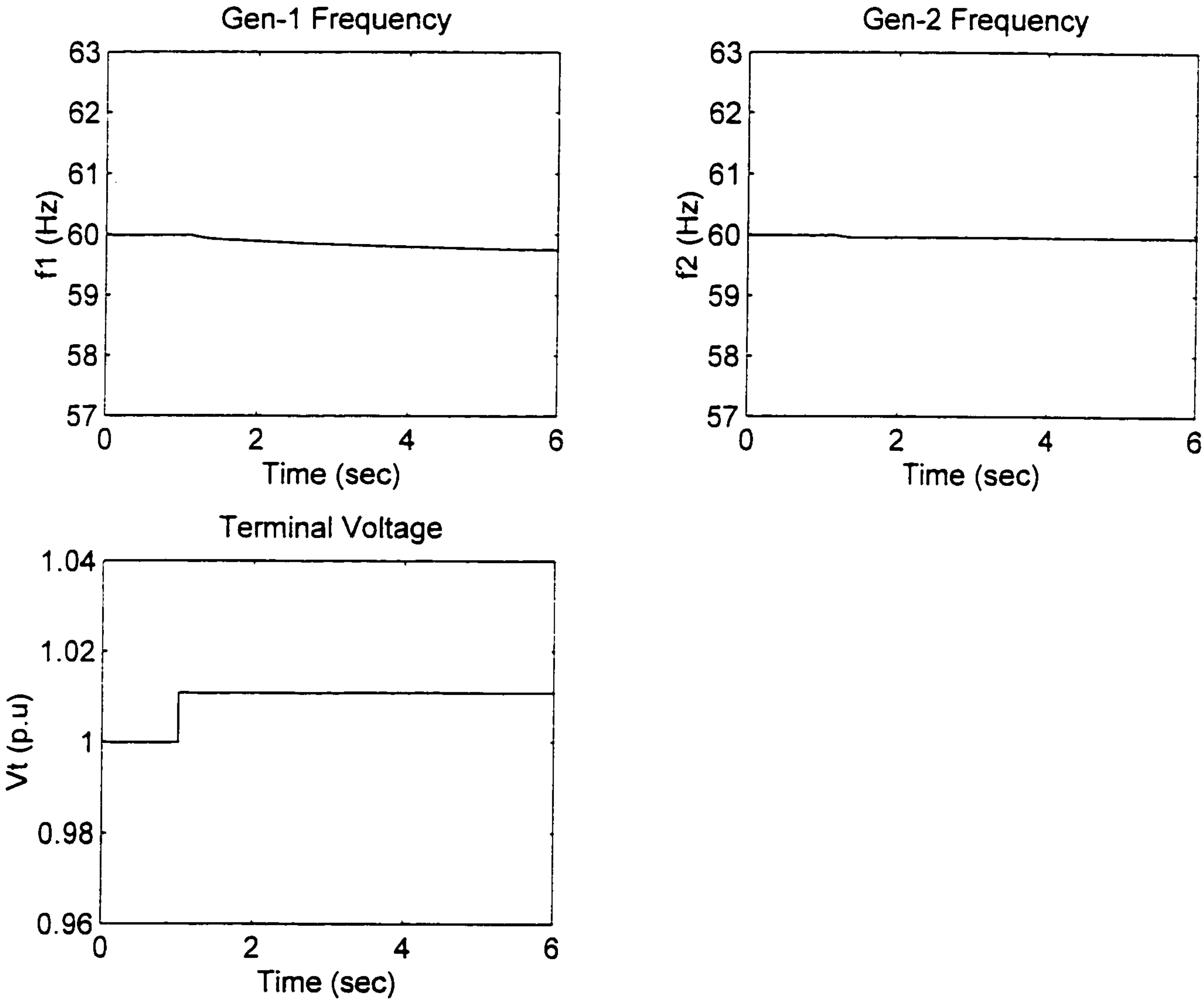

Fig 5.9 Parallel generators response to a step change in load active power, with weighted Q & R matrices LQR control. Case # 5: weight # 13, 14, & 16 are set to zero

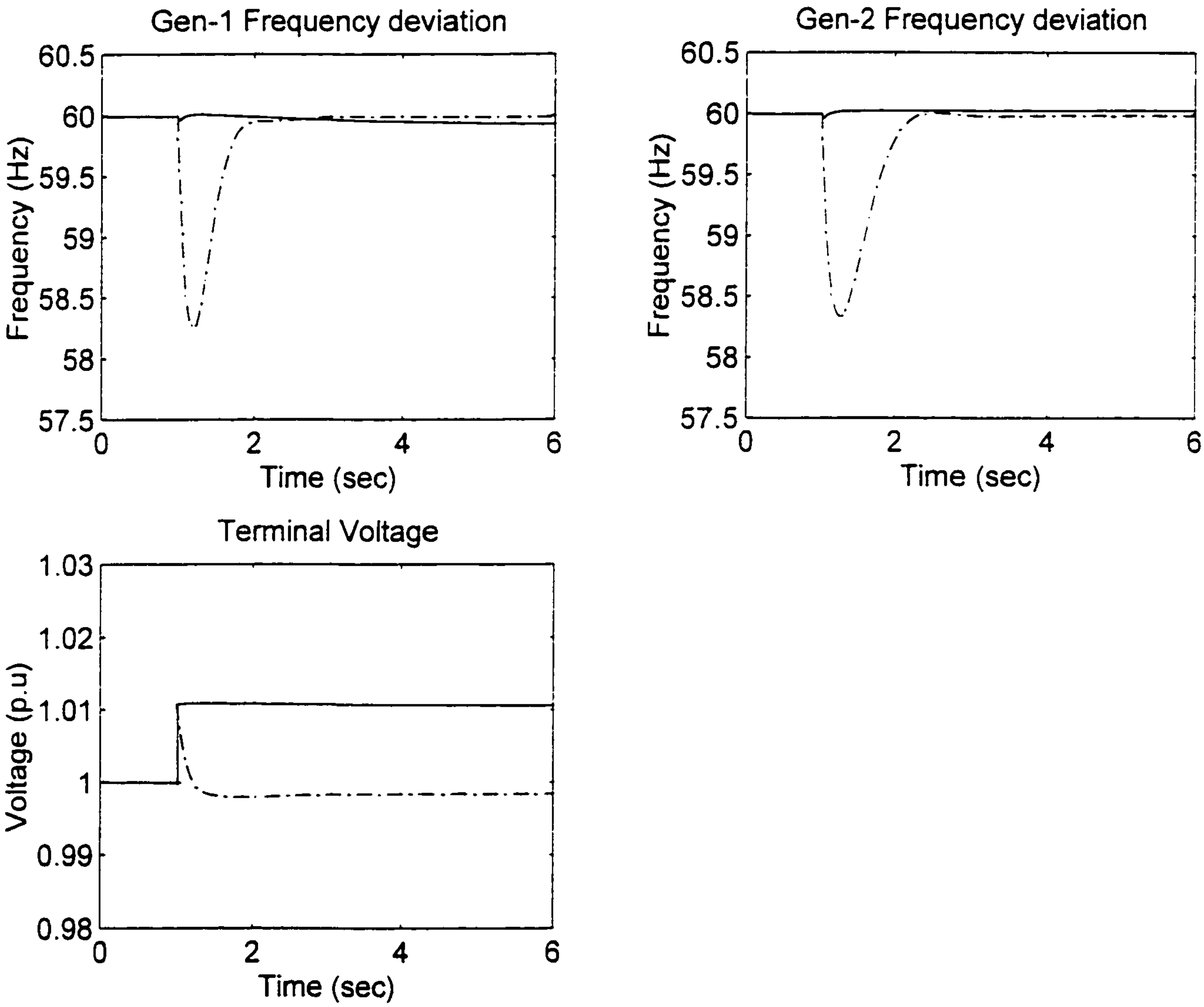

Fig 5.10 Comparative parallel generators response to a step change in load active power Standard centralized LQR Versus weighted Q & R matrices LQR control.
(- - - with centralized LQR —— with calculated Q&R LQR)

Chapter 6

CONCLUSION & RECOMMENDATIONS

6.1 Conclusion:

As mentioned in the introduction, this thesis is one step towards having complete modeling and control of diesel driven generator set and examining its behavior under different load perturbations, using traditional controls available in the industry nowadays. It was unfortunate that the subject of the thesis was not taken into the depth I wished, however the work implemented in the thesis will be a good background work for the future researchers in this subject.

The stand-alone diesel generator was modeled using 3^{rd} order model then linearized around its initial operating parameters. The linearized model gave very closed results to the nonlinear model. It was noticed that for stand-alone diesel generators when a resistive load is suddenly applied the voltage tends to rise and not to dip as it was thought. While on reactive sudden load changes, the voltage will dip.

It was noticed that the generator active and reactive powers are functions of the machines internal voltage E_q' . Not like the case of the generator connected to infinite bus-bar, where the powers are functions of both the E_q' as well as the rotor angle δ. It was also noticed that the rotor angle in the infinite bus-bar case is totally different from what we called rotor-to-stator angle of our study case. In the stand-alone generator case this angle is algebraically calculated.

In the parallel operation case, the governors and excitation systems of the parallel generators are controlling the speeds and terminal voltages of the two machines respectively, to the input reference settings, where the load itself controls the output powers from the generators. Not like in the infinite bus case, where the role of the governors and excitation systems is to control the output active and reactive powers of the generators leaving the control of the frequency and terminal voltage to the grid, where these two are considered to be always constant.

It was clearly shown that the amount of change in active load power is added to the rotor shaft, tending to accelerate or decelerate it leaving the governor to correct the dip or rise in the frequency. Similarly any change in the load reactive power will be scaled and directly added to the excitation filed voltage tending to increase or decrease the E_q' leaving the voltage regulator to correct the transient rise or dip in the terminal voltage. This is also different from the infinite bus case.

It can also be concluded that LQR methods are useful and can improve the transient responses of the frequency and terminal voltage of the generators during sudden load changes, and it is clear that when the diesel generator are operating isochronously in parallel and sharing common load the decentralized controls are not able to stabilize them, and it is only the centralized control who are able to stabilize and control such systems.

The LQR method using calculated Q and R matrices was tested on this system. It was clear that the method is very effective in supressing the transients of the system. But it was noticed that it generates steady state error which requires integrator to eliminate.

6.2 Recommendations:

1. It is strongly recommend that this subject be studied very well, with complete funding from the local diesel generators manufacturers, as there is still lot of room for system modeling improvement and development, specially by knowing that the performance and cost are very critical criteria, in addition to the tough competition in the market between the manufacturers of the diesel generators and associated control systems.
2. Complete nonlinear modeling of the diesel generator must be carried out taking into consideration the diesel engine dynamics, torsional torques and turbocharger effects. Also new engines with electronic injection systems shall be modeled. In these new engines the governor actuator is substituted by fuel solenoid distributed on all engine cylinders and these are all controlled typically from a microprocessor and a digital PID loop taking into consideration all diesel engine operating parameters.
3. More than 50% of diesel generator exciters are equipped with PMG (Permanent Magnet Generator). This should be modeled and included in the dynamics of the exciter and field equations.

4. New generators have been introduced to the markets recently using special auxiliary windings in the stator and rotor to improve the electrical transient response of the machines to sudden disturbances such as heavy motor starting. These have to be modeled and studied.
5. The synchronization control should be considered in all future studies. The effect of such was clearly shown in the parallel cases specially the non-linear one where one of the generators tends to go with its own frequency while the other one remains as it is.
6. In future studies it is recommended to take into consideration the dynamic load and non-linear load models instead of static loads. This should cover all motor loads as well as other loads.
7. Generator models can be extracted from real experimental tests using identification methods. Generator manufacturers and universities must cooperate to implement this.
8. Modern microcontrollers shall be used with multi digital and analogue input and output channels to control parallel operating generators.
9. New control algorithms must be tried to replace all traditional controls used in speed governing systems.
10. Extend the modeling of parallel operation case to more than two generators, to be applicable to N number of generator sets.
11. Further research is needed on the LQR control using the calculated Q and R matrices to get better results with minimum control effort. Special emphasis is required on how to calculate Q matrix as it was noticed that it could increase the steady state error. More work is needed in this direction.

12. Other modern control algorithms need to be investigated on the parallel diesel generator system and tested. This could be good future work..

Appendix-A

SIMULATION DATA

A.1. Synchronous Machines Data:

A.1.1 Moment of Inertia:

Table A.1 lists typical low voltage alternators (*Marathon Electric*) mechanical data, where moment of inertia or WK^2 is given in lb.ft^2. MVA ratings and weights are also shown.

Model Number	RATING MVA	Voltage V	WK^2 (lb-ft2)	Weight Lbs (Kg)
432RSL4013	0.325	208Y-240Y/416Y	69.4	1600(720)
433RSL4019	0.453	208Y-240Y/416Y	113.3	2340(1053)
572RSL4024	0.544	208Y-240Y/416Y	204.8	2730(1229)
573RSL4032	0.813	208Y-240Y/416Y	265.5	3400(1530)
574RSL4036	0.906	208Y-240Y/416Y	322.3	4080(1836)
740RSL4046	1.275	416Y-480Y	403.8	5200(2340)
742RSL4048	1.375	416Y-480Y	656.2	6300(2835)
743RSL4050	1.588	416Y-480Y	749.1	7230(3254)
744RSL4054	2.100	416Y-480Y	937.2	8600(3870)

Table A.1 Typical Alternators Rotor WK^2 and weights (Source: *Marathon Electric*)

A.1.2. Synchronous Machines Parameters:

Table A.2 lists the two generators parameters used in the thesis model test (DG-1 and DG-2). The parameters shown are referred to each generator base. Where Table A.3 refers DG-2 parameters to DG-1 base.

Machine Parameters	Symbol	Unit	DG-1	DG-2
Direct Axis synchronous Reactance	X_d	p.u.	1.518	1.328
Quadrature Axis Synchronous Reactance	X_q	p.u.	0.865	0.797
Direct Axis Transient Reactance	X_d'	p.u.	0.18	0.192
Quadrature axis Transient Reactance	X_q'	p.u.	0.695	0.767
Direct Axis Subtransient Reactance	X_d''	p.u.	0.151	0.176
Open Circuit Transient Time Constant	τ_{do}'	Sec	3.267	3.446
Moment of Inertia	WK^2	$lb.ft^2$	757.5	965
Inertia Constant	M	MJ/MVA	0.4123	0.425
Base MVA	S_{mach}	MVA	1.375	1.7

Table A.2 DG-1 and DG-2 Synchronous Machine Reactances and Constants referred to each generator base.

Machine Parameters	Symbol	Units	DG-1	DG-2
Direct Axis synchronous Reactance	X_d	p.u.	1.518	1.0741
Quadrature Axis Synchronous Reactance	X_q	p.u.	0.865	0.6446
Direct Axis Transient Reactance	X_d'	p.u.	0.18	0.1553
Quadrature axis Transient Reactance	X_q'	p.u.	0.695	0.5468
Direct Axis Subtransient Reactance	X_d''	p.u.	0.151	0.1423
Open Circuit Transient Time Constant	T_{do}'	sec	3.267	3.446
Moment of Inertia	WK^2	lb.ft2	757.5	965
Inertia Constant	H	MJ/MVA	0.4123	0.5253
Base MVA	S_{mach}	MVA	1.375	1.375

Table A.3 Generators DG-1 and DG-2 Excitation system Gain and Time constants referred to DG-1 base.

A.2. Excitation Controls Data:

Table A.4 details the gains and time constants of the DG-1 and DG-2 voltage regulators and and excitation system.

Excitation Parameters	Symbol	Units	DG-1	DG-2
Voltage Regulator Gain Constant	K_A	p.u.	400	400
Maximum Voltage Regulator Output	V_R (max)	p.u.	8	8
Minimum Voltage Regulator Output	V_R (min)	p.u.	0	0
Exciter Self Excitation Gain Constant	KE	p.u.	1	1
Exciter Time Constant	TE	sec	0.08	1
AVR Stabilizing Circuit Gain Constant	KF	p.u.	0.005	0.008
AVR Stabilizing Circuit Time Constant	TF	sec	0.6	0.5
Rotating Exciter Saturation Factor	SE	p.u.	0.3	0.4

Table A.4 DG-1 and DG-2 Excitation System gains and Time Constants

A.3. Diesel Engine Typical Parameters:

Table A.5 below lists some typical diesel engine and fuel actuator time constants and gains for the purpose of simulation studies of this thesis.

Parameter	Definition	Units	Value
K_1	Engine Torque Constant	p.u.	0.8 - 1.5
K_2	Actuator Constant	p.u.	1
K_3	Current Driver Constant	p.u.	1
T_1	Engine Dead Time	Sec	0 - 0.25
T_2	Actuator Time Constant	Sec	0.05 - 0.20

Table A.5 Typical Engine and Actuator gain & time constants

A.4. Typical Speed Controller Parameters:

Table A.6 below lists some typical PID type engine speed governor controller gain constants for this thesis simulation purposes.

Speed Controller Parameters	Symbol	Units	DG-1	DG-2
Proportional Gain Constant	K_G	p.u.	5	5
Integral Gain Constant	K_{IN}	p.u.	16	16
Derivative Gain Constant	K_D	p.u.	0	0
Engine Rated Speed	ω_o	p.u.	1	1

Table A.6 PID Type Speed Controller Constants

Appendix-B

STAND-ALONE GENERATOR

B.1 Initial States Values and Loading Conditions:

$[\omega_o, \delta_o, Eq'_o, E_{fd_0}, V_{R_0}, V_{F_0}, X_{A_0}, X_{G_0}, P_{m_0}]^T$

The diesel generator (DG-1) was initially operating at the following loading conditions:

$P_{e_0} = 0.4$ p.u. $Q_{e_0} = 0.3$ p.u PF = 0.8 lagging $V_{t_0} = 1.0$ p.u

$$V_{d_0} = P_{e_0}V_{t_0}/[\sqrt{(P_{e_0}{}^2+(Q_{e_0}+V_{t_0}{}^2/x_q)^2}] \quad \text{(B.1)}$$

$$I_{q_0} = V_{d_0}/X_q \quad \text{(B.2)}$$

$$V_{q_0} = \sqrt{V_{t_0}{}^2 - V_{d_0}{}^2} \quad \text{(B.3)}$$

$$I_{d_0} = (P_{e_0} - I_{q_0}V_{q_0})/V_{d_0} \quad \text{(B.4)}$$

$$E'_{q_0} = V_{q_0} + x_d' I_{d_0} \quad \text{(B.5)}$$

$$\delta_o = \sin^{-1}(V_{d_0}/V_{t_0}) \quad \text{(B.6)}$$

$$E_{fd_0} = E_q{}'_0 + (x_d - x_d')I_{d_0} \quad \text{(B.7)}$$

$$V_{Ro} = (K_E + S_E)E_{fd_0} \quad \text{(B.8)}$$

$$V_{REF} = E_{fd_0}/K_A + V_{to} \quad \text{(B.9)}$$

R_L and X_L of the load are calculated from P_L, Q_L and terminal voltage V_t as follows:

$$R_L = V_t^2 P_L/(P_L^2 + Q_L^2) \quad \text{(B.10)}$$

$$X_L = V_t^2 Q_L/(P_L^2 + Q_L^2) \quad \text{(B.11)}$$

B.2 Dynamic Equation Coefficients:

The speed controller nonlinear equation coefficients are defined as,

$m_1 = K_G K_P/M - K_P K_D (2K_q/\tau_{do}' + D/M)/M$ (B.12)

$m_2 = 2K_P K_D/(M\tau_{do}')$ (B.13)

$m_3 = K_G D/M - K_{IN} - K_D D_2/(M_2)$ (B.14)

$m_4 = K_D D/M^2 - K_G/M + 2K_D/(M\tau_1)$ (B.15)

$m_5 = K_1 K_D/(M\tau_2) + 2K_1 K_D/(M\tau_1)$ (B.16)

$m_6 = K_1 K_D K_2 K_3/(M\tau_2)$ (B.17)

The speed controller linearized equation coefficients are defined as,

$d_1 = K_G D/M - K_{IN} - D^2 K_D/(M^2)$ (B.18)

$d_2 = (2K_G K_{Po} E_{q\,o}' - 2K_D D K_{Po} E_{q\,o}')/M + (2K_D K_{Po} E_{fdo} - 4K_D K_{Po} K_{qo} E_{q\,o}')/(M\tau_{do}')$ (B.19)

$d_3 = 2K_D K_{Po} E_{qo}'/(M\tau_{do}')$ (B.20)

$d_4 = (K_G E_{qo}'^2 - D K_D E_{qo}'^2)/M + (2K_D E_{qo}' E_{fdo} - 2K_D K_{qo} E_{qo}'^2)/(M\tau_{do}')$ (B.21)

$d_5 = 2K_D E_{qo}'^2 K_{Po}/(M\tau_{do}')$ (B.22)

$d_6 = K_1 K_D/(MT_2) + 2K_1 K_D/(MT_1)$ (B.23)

$d_7 = K_1 K_2 K_3 K_D/(MT_2)$ (B.24)

$d_8 = K_D D/(M^2) - K_G/M + 2K_D/(MT_1)$ (B.25)

$\Delta K_W = d_4 \Delta K_P - d_5 \Delta K_q$ (B.26)

$\Delta K_P = K_P - K_{PO}$ (B.27)

$\Delta K_Q = K_Q - K_{QO}$ (B.28)

$\Delta K_V = K_V - K_{VO}$ (B.29)

$\Delta K_q = K_q - K_{Qo}$ (B.30)

Appendix-C

PARALLEL GENERATORS

C.1 Initial States Values and Loading Conditions:

The initial state value vector for the two parallel generators model:

$[\omega_{o_1}\ \delta_{o_1}\ E_{qo_1}'\ E_{fdo_1}\ V_{Ro_1}\ V_{Fo_1}\ X_{Ao_1}\ X_{Go_1}\ P_{mo_1}\ \omega_{o_2}\ \delta_{o_2}\ E_{qo_2}\ E_{fdo_2}\ V_{Ro_2}\ V_{Fo_2}\ X_{Ao_2}\ X_{Go_2}\ P_{mo_2}]^T$

The two diesel generators (DG-1 and DG-2) were initially operating in parallel and connected to a local load:

$P_{L_o} = 0.7272$ p.u. $Q_{L_o} = 0.5454$ p.u PF = 0.8 lagging $V_{t_o} = 1.0$ p.u

Where:

$P_{eo_1} = 0.325 \qquad P_{eo_2} = 0.402$

$V_{do} = P_{L_o}V_{t_o}/[\sqrt{(P_{Lo}^2+(Q_{Lo}+V_{to}^2(1/x_{q_1}+1/x_{q_2}))^2}]$ (C.1)

$I_{qo_1} = V_{d_o}/x_{q_1}$ (C.2)

$I_{qo_2} = V_{d_o}/x_{q_2}$ (C.3)

$V_{q_o} = \sqrt{(V_{t_o}^2 - V_{d_o}^2)}$ (C.4)

$I_{do_1} = (P_{eo_1} - I_{qo_1}V_{q_o})/V_{d_o}$ (C.5)

$I_{do_2} = (P_{eo_2} - I_{qo_2}V_{qo})/V_{do}$ (C.6)

$E_{qo_1}' = V_{q_o} + x_{d_1}'I_{do_1}$ (C.7)

$E_{qo_2}' = V_{q_o} + x_{d_2}'I_{do_2}$ (C.8)

$\delta_{o_1} = \sin^{-1}(V_{d_o}/V_{t_o})$ (C.9)

$\delta_{o_2} = \delta_{o_1}$ (C.10)

$E_{fdo_1} = E_{qo_1}' + (x_{d_1} - x_{d_1}')I_{do_1}$ (C.11)

$E_{fdo_2} = E_{q0_2}'+(x_{d_2}-x_{d_2}')I_{do_2}$ (C.12)

$V_{Ro_1} = (K_{E_1}+S_{E_1})E_{fdo_1}$ (C.13)

$V_{Ro_2} = (K_{E_2}+S_{E_2})E_{fdo_2}$ (C.14)

$P_{Mo_1} = P_{o_1}+D\omega_{o_1}$ (C.15)

$P_{Mo_2} = P_{o_2}+D\omega_{o_2}$ (C.16)

$V_{Fo_1} = (-K_{A_1}V_{to}-VR_{o_1}+K_{A_1}V_{REF_1}/K_{A_1}$ (C.17)

$V_{Fo_2} = (-K_{A_2}V_{to}-V_{Ro_2}+K_{A_2}V_{REF_2})/K_{A_2}$ (C.18)

$X_{Go_1} = P_{Mo_1}/(K_{1_1}K_{2_1}K_{3_1})$ (C.19)

$X_{Go_2} = P_{Mo_2}/(K_{1_2}K_{2_2}K_{3_2})$ (C.20)

$X_{Ao_1} = K_{2_1}K_{3_1}X_{Go_1}$ (C.21)

$X_{Ao_2} = K_{2_2}K_{3_2}X_{Go_2}$ (C.22)

$V_{REF_1} = E_{fdo_1}/K_{A_1}+V_{to}$ (C.23)

$V_{REF_2} = E_{fdo_2}/K_{A_2}+V_{to}$ (C.24)

C.2 Network Equation Coefficients:

$X_1 = x_{q1}+x_L+x_Lx_{q1}/x_{q2}$ (C.25)

$X_2 = R_L+R_Lx_{d1}'/x_{d2}'$ (C.26)

$X_3 = (R_L/x_{d2}')E_{q2}'-(R_L/x_{d2}')E_{q1}'$ (C.27)

$X_4 = E_{q1}'-(x_L/x_{d2}')E_{q2}'+(x_L/x_{d2}')E_{q1}'$ (C.28)

$X_5 = x_{d1}'+x_L+x_Lx_{d1}'/x_{d2}'$ (C.29)

$X_6 = R_L+(R_L/x_{q2})x_{q1}$ (C.30)

$X_7 = X_1/(X_1X_5+X_2X_6)$ (C.31)

$X_8 = X_6/(X_1X_5+X_2X_6)$ (C.32)

$X_9 = X_7+X_7X_L/x_{d2}'+X_8R_L/x_{d2}'$ (C.33)

$X_{10} = X_7X_L/x_{d2}'+X_8R_L/x_{d2}'$ (C.34)

$X_{11} = (1/X_6)(1+x_L/x_{d2}'-X_5X_9)$ (C.35)

$X_{12} = (1/X_6)(X_5X_{10}-x_L/x_{d2}')$ (C.36)

$X_{13} = x_{q1}X_{11}/x_{q2}$ (C.37)

$X_{14} = x_{q1}X_{12}/x_{q2}$ (C.38

$X_{15} = (1/x_{d2}')(x_{d1}'X_9-1)$ (C.39)

$$X_{16} = (1/x_{d2}')(1-x_{d1}'X_{10}) \tag{C.40}$$

$$X_{17} = x_{q1}X_{11} \tag{C.41}$$

$$X_{18} = x_{q1}X_{12} \tag{C.42}$$

$$X_{19} = 1-x_{d1}'X_9 \tag{C.43}$$

$$X_{20} = x_{d1}'X_{10} \tag{C.44}$$

$$X_{21} = X_{17}X_9+X_{19}X_{11} \tag{C.45}$$

$$X_{22} = X_{18}X_9-X_{17}X_{10}+X_{19}X_{12}+X_{20}X_{11} \tag{C.46}$$

$$X_{23} = X_{20}X_{12}-X_{18}X_{10} \tag{C.47}$$

$$X_{24} = X_{17}X_{15}+X_{19}X_{13} \tag{C.48}$$

$$X_{25} = X_{17}X_{16}+X_{18}X_{15}+X_{19}X_{14}+X_{20}X_{13} \tag{C.49}$$

$$X_{26} = X_{18}X_{16}+X_{20}X_{14} \tag{C.50}$$

$$X_{27} = x_{d1}'X_9-x_{d1}X_9-1 \tag{C.51}$$

$$X_{28} = x_{d1}X_{10}-x_{d1}'X_{10} \tag{C.52}$$

$$X_{29} = x_{d2}'X_{15}-x_{d2}X_{15} \tag{C.53}$$

$$X_{30} = x_{d2}'X_{16}-x_{d2}X_{16}-1 \tag{C.54}$$

$$X_{31} = 2X_{21}X_{27}/\tau_{do1}' \tag{C.55}$$

$$X_{32} = 2X_{21}X_{28}/\tau_{do1}'+X_{22}X_{27}/\tau_{do1}'+X_{22}X_{30}/\tau_{do2}'+2X_{23}X_{29}/\tau_{do2}' \tag{C.56}$$

$$X_{33} = X_{22}X_{28}/\tau_{do1}'+2X_{23}X_{30}/\tau_{do2}' \tag{C.57}$$

$$X_{34} = 2X_{21}/\tau_{do1}' \tag{C.58}$$

$$X_{35} = X_{22}/\tau_{do1}' \tag{C.59}$$

$$X_{36} = X_{22}/\tau_{do2}' \tag{C.60}$$

$$X_{37} = 2X_{23}/\tau_{do2}' \tag{C.61}$$

$$X_{38} = 2X_{24}X_{27}/\tau_{do1}'+X_{25}X_{29}/\tau_{do2}' \tag{C.62}$$

$$X_{39} = 2X_{24}X_{28}/\tau_{do1}'+X_{25}X_{27}/\tau_{do1}'+X_{25}X_{30}/\tau_{do2}'+2X_{26}X_{29}/\tau_{do2}' \tag{C.63}$$

$$X_{40} = X_{25}X_{28}/\tau_{do1}'+2X_{26}X_{30}/\tau_{do2}' \tag{C.64}$$

$$X_{41} = 2X_{24}/\tau_{do1}' \tag{C.65}$$

$$X_{42} = X_{25}/\tau_{do1}' \tag{C.66}$$

$$X_{43} = X_{25}/\tau_{do2}' \tag{C.67}$$

$$X_{44} = 2X_{26}/\tau_{do2}' \tag{C.68}$$

$$X_{45} = X_9X_{19}-X_{11}X_{17} \tag{C.69}$$

$$X_{46} = X_9X_{20}-X_{10}X_{19}-X_{11}X_{18}-X_{12}X_{17} \tag{C.70}$$

$$X_{47} = X_{10}X_{20}+X_{12}X_{18} \tag{C.71}$$

$$X_{48} = X_{15}X_{19}-X_{13}X_{17} \tag{C.72}$$

$$X_{49} = X_{16}X_{19}+X_{15}X_{20}-X_{14}X_{17}-X_{13}X_{18} \tag{C.73}$$

$$X_{50} = X_{16}X_{20}-X_{14}X_{18} \tag{C.74}$$

C.3 Dynamic Equation Coefficients:

The speed controllers nonlinear equations coefficients are defined as,

$$m_{11} = K_{G1}D/M_1-K_{IN1}-K_{D1}D^2/M_1^2 \tag{C.75}$$

$$m_{21} = K_{G1}/M_1-K_{D1}D/M_1^2 \tag{C.76}$$

$$m_{31} = 2K_{D1}/(M_1\tau_{11})-K_{G1}/M_1+K_{D1}D/M_1^2 \tag{C.77}$$

$$m_{41} = K_{D1}/M_1 \tag{C.78}$$

$$m_{51} = (K_{D1}/M_1)(K_{11}/\tau_{21}+2K_{11}/\tau_{11}) \tag{C.79}$$

$$m_{61} = K_{11}K_{D1}K_{21}K_{31}/(M_1\tau_{21}) \tag{C.80}$$

$$m_{12} = K_{G2}D/M_2-K_{IN2}-K_{D2}D^2/M_2^2 \tag{C.81}$$

$$m_{22} = K_{G2}/M_2-K_{D2}D/M_2^2 \tag{C.82}$$

$$m_{32} = 2K_{D2}/(M_2\tau_{12})-K_{G2}/M_2+K_{D2}D/M_2^2 \tag{C.83}$$

$$m_{42} = K_{D2}/M_2 \tag{C.84}$$

$$m_{52} = (K_{D2}/M_2)(K_{12}/\tau_{22}+2K_{12}/\tau_{12}) \tag{C.85}$$

$$m_{62} = K_{12}K_{D2}K_{22}K_{32}/(M_2\tau_{22}) \tag{C.86}$$

While the linearized speed controller equations coefficients are defined as,

$$d_{11} = K_{G1}D/M_1-K_{IN1}-K_{D1}D^2/M_1^2 \tag{C.87}$$

$$d_{21} = K_{G1}K_{P11}/M_1-K_{D1}DK_{P11}/M_1^2+K_{D1}K_{P11}X_{27i}/(M_1\tau_{do1}')+K_{D1}K_{P12}X_{29i}/(M_1\tau_{do2}') \tag{C.88}$$

$$d_{31} = K_{P12}K_{G1}/M_1-K_{D1}DK_{P12}/M_1^2+K_{D1}K_{P11}X_{28i}/(M_1\tau_{do1}')+K_{D1}K_{P12}X_{30i}/(M_1\tau_{do2}') \tag{C.89}$$

$$d_{41} = K_{D1}K_{P11}/(M_1\tau_{do1}') \tag{C.90}$$

$$d_{51} = K_{D1}K_{P12}/(M_1\tau_{do2}') \tag{C.91}$$

$$d_{61} = (K_{D1}/M_1)(K_{11}/\tau_{21}+2K_{11}/\tau_{11}) \tag{C.92}$$

$$d_{71} = K_{11}K_{D1}K_{21}K_{31}/(M_1\tau_{21}) \tag{C.93}$$

$$d_{81} = 2K_{D1}/(M_1\tau_{21})+K_{D1}D/M_1^2-K_{G1}/M_1 \tag{C.94}$$

$$\Delta K_{W1} = (K_{D1}/M_1)(-D\Delta K_{P1}/M_1+K_{P11}\Delta K_{q1}D/\tau_{do1}'+K_{P12}\Delta K_{q2}/\tau_{do2}')+K_{G1}\Delta K_{P1}/M_1 \tag{C.95}$$

$$d_{12} = K_{G2}D/M_2-K_{IN2}-K_{D2}D^2/M_2^2 \tag{C.96}$$

$$d_{22} = K_{G2}K_{P21}/M_2-K_{D2}DK_{P21}/M_2^2+K_{D2}K_{P21}X_{27i}/(M_2\tau_{do1}')+K_{D2}K_{P22}X_{29i}/(M_2\tau_{do2}') \tag{C.97}$$

$$d_{32} = K_{P22}K_{G2}/M_2 - K_{D2}DK_{P22}/M_2^2 + K_{D2}K_{P21}X_{28i}/(M_1\tau_{do1}') + K_{D2}K_{P22}X_{30i}/(M_2\tau_{do2}') \quad (C.98)$$

$$d_{42} = K_{D2}K_{P21}/(M_2\tau_{do1}') \quad (C.99)$$

$$d_{52} = K_{D2}K_{P22}/(M_2\tau_{do2}') \quad (C.100)$$

$$d_{62} = (K_{D2}/M_2)(K_{12}/\tau_{22} + 2K_{12}/\tau_{12}) \quad (C.101)$$

$$d_{72} = K_{12}K_{D2}K_{22}K_{32}/(M_2\tau_{22}) \quad (C.102)$$

$$d_{82} = 2K_{D2}/(M_2\tau_{22}) + K_{D2}D/M_2^2 - K_{G2}/M_2 \quad (C.103)$$

$$\Delta K_{W2} = (K_{D2}/M_2)(-D\Delta K_{P2}/M_2 + K_{P21}\Delta K_{q2}D/\tau_{do1}' + K_{P22}\Delta K_{q2}/\tau_{do2}') + K_{G2}\Delta K_{P2}/M_2 \quad (C.104)$$

C.4 Linearized Dynamic Equations:

For two parallel diesel generators the swing equations are linearized as,

$$\Delta\dot{\omega}_1 = (1/M_1)[\Delta P_{m_1} - \Delta P_{e_1}]$$

$$= (1/M_1)[\Delta P_{m_1} - K_{P1_1}\Delta E_{q_1}' - K_{P1_2}\Delta E_{q_2}' - \Delta K_{P_1}] \quad (C.105)$$

$$\Delta\dot{\omega}_2 = (1/M_2)[\Delta P_{m_2} - \Delta P_{e_2} - \Delta P_{D_2}]$$

$$= (1/M_2)[\Delta P_{m_2} - K_{P2_1}\Delta E_{q_1}' - K_{P2_2}\Delta E_{q_2}' - \Delta K_{P_2}] \quad (C.106)$$

and the rotor angle equations are linearized as

$$\Delta\dot{\delta}_1 = \omega_o\Delta\omega_1 \quad (C.107)$$

$$\Delta\dot{\delta}_2 = \omega_o\Delta\omega_2 \quad (C.108)$$

The internal voltage equations (3.24) and (3.25) are linearized as,

$$\Delta\dot{E}_{q_1}' = (1/\tau_{do_1}')(\Delta E_{fd_1} + X_{27i}\Delta E_{q_1}' + X_{28i}\Delta E_{q_2}' + \Delta K_{q_1}) \quad (C.109)$$

$$\Delta\dot{E}_{q_2}' = (1/\tau_{do_2}')(\Delta E_{fd_2} + X_{29i}\Delta E_{q_1}' + X_{30i}\Delta E_{q_2}' + \Delta K_{q_2}) \quad (C.110)$$

The excitation systems equations (3.26), (3.27), (3.28), (3.29), (3.30) and (3.31) are linearized as,

$$\Delta\dot{V}_{R_1} = -(K_{A_1}\Delta V_t)/T_{A_1} - \Delta V_{R_1}/T_{A_1} - (K_{A_1}\Delta V_{F_1})/T_{A_1} + (K_{A_1}\Delta V_{REF_1})/T_{A_1} \quad (C.111)$$

$$\Delta\dot{E}_{fd_1} = \Delta V_{R_1}/T_{E_1} - (K_{E_1} + S_{E_1})\Delta E_{fd_1}/T_{E_1} \quad (C.112)$$

$$\Delta\dot{V}_{F_1} = -(K_{A_1}K_{F_1})\Delta V_t/(T_{A_1}T_{F_1}) - (K_{A_1}K_{F_1} + T_{A_1})\Delta V_{F_1}/(T_{A_1}T_{F_1}) + K_{A_1}K_{F_1}\Delta V_{REF_1}/(T_{A_1}T_{F_1}) -$$

$$K_{F_1}\Delta V_{R_1}/(T_{A_1}T_{F_1}) \tag{C.113}$$

$$\Delta \dot{V}_{R_2} = -(K_{A_2}\Delta V_t)/T_{A_2} - \Delta V_{R_2}/T_{A_2} - (K_{A_2}\Delta V_{F_2})/T_{A_2} + (K_{A_2}\Delta V_{REF_2})/T_{A_2} \tag{C.114}$$

$$\Delta \dot{E}_{fd_2} = \Delta V_{R_2}/T_{E_2} - (K_{E_2}+S_{E_2})\Delta E_{fd_2}/T_{E_2} \tag{C.115}$$

$$\Delta \dot{V}_{F_2} = -(K_{A_2}K_{F_2})\Delta V_t/(T_{A_2}T_{F_2}) - (K_{A_2}K_{F_2}+T_{A_2})\Delta V_{F_2}/(T_{A_2}T_{F_2}) + K_{A_2}K_{F_2}\Delta V_{REF_2}/(T_{A_2}T_{F_2})$$

$$- K_{F_2}\Delta V_{R_2}/(T_{A_2}T_{F_2}) \tag{C.116}$$

The diesel engines and actuators equations (3.33), (3.34), (3.35) and (3.36) are linearized as,

$$\Delta \dot{X}_{A_1} = -(1/\tau_{2_1})\Delta X_{A_1} + (K_{2_1}K_{3_1}/\tau_{2_1})\Delta X_{G_1} \tag{C.117}$$

$$\Delta \dot{X}_{A_2} = -(1/\tau_{2_2})\Delta X_{A_2} + (K_{2_2}K_{3_2}/\tau_{2_2})\Delta X_{G_2} \tag{C.118}$$

$$\Delta \dot{P}_{m_1} = -(2/\tau_{1_1})\Delta P_{m_1} + [K_{1_1}/\tau_{2_1}+2K_{1_1}/\tau_{1_1}]\,\Delta X_{A_1} - (K_{1_1}K_{2_1}K_{3_1}/\tau_{2_1})\,\Delta X_{G_1} \tag{C.119}$$

$$\Delta \dot{P}_{m_2} = -(2/\tau_{1_2})\Delta P_{m_2} + [K_{1_2}/\tau_{2_2}+2K_{1_2}/\tau_{1_2}]\,\Delta X_{A_2} - (K_{1_2}K_{2_2}K_{3_2}/\tau_{2_2})\,\Delta X_{G_2} \tag{C.120}$$

While the speed controllers equations (3.37) and (3.38) are linearized as,

$$\Delta \dot{X}_{G_1} = d_{1_1}\Delta\omega_1+d_{2_1}\Delta E_{q_1}'+d_{3_1}\Delta E_{q_2}'+d_{4_1}\Delta E_{fd_1}+d_{5_1}\Delta E_{fd_2}\; d_{6_1}X_{A_1}+d_{7_1}\,\Delta X_{G_1} + d_{8_1}\,\Delta P_{m_1}+\Delta K_{W_1}$$

$$+K_{IN_1}\Delta\omega_{REF_1} \tag{C.121}$$

$$\Delta \dot{X}_{G_2} = d_{1_2}\Delta\omega_2+d_{2_2}\Delta E_{q_1}'+d_{3_2}\Delta E_{q_2}'+d_{4_2}\Delta E_{fd_1}+d_{5_2}\Delta E_{fd_2}\; d_{6_2}X_{A_2}+d_{7_2}\,\Delta X_{G_2} + d_{8_2}\,\Delta P_{m_2}+\Delta K_{W_2}$$

$$+K_{IN_2}\Delta\omega_{REF_2}$$

C.5 Auxiliary Equations:

Substituting V_d, V_q, I_{d_1}, I_{d_2}, I_{q_1} and I_{q_2} from equations (3.18), (3.19), (3.20), (3.21), (3.22) and (3.23) in P_{e_1} and P_{e_2} equations (3.5) and (3.6) we write P_{e_1} , P_{e_2} as,

$$P_{e_1} = X_{21}E_{q_1}'^2+X_{22}E_{q_1}'E_{q_2}'+X_{23}E_{q_2}'^2 \tag{C.122}$$

$$P_{e_2} = X_{24}E_{q_1}'^2+X_{25}E_{q_1}'E_{q_2}'+X_{26}E_{q_2}'^2 \tag{C.123}$$

Similarly we write Q_{e_1} and Q_{e_2} as,

$$Q_{e_1} = X_{45}E_{q_1}'^2+X_{46}E_{q_1}'E_{q_2}'-X_{47}E_{q_2}'^2 \tag{C.124}$$

$$Q_{e_2} = X_{48}E_{q_1}'^2+X_{49}E_{q_1}'E_{q_2}'+X_{50}E_{q_2}'^2 \tag{C.125}$$

Where X_{21} , X_{22} , X_{23} , X_{24} , X_{25} , X_{26} , X_{45} , X_{46} , X_{47} , X_{48} , X_{49} and X_{50} are all constants depending on generator parameters and loading conditions as defined in Appendix-C.2

The above electrical power equations are linearized to get,

$$\Delta P_{e_1} = K_{P1_1}\Delta E_{q_1}'+K_{P1_2}\Delta E_{q_2}'+\Delta K_{P_1} \quad (C.126)$$

$$\Delta P_{e_2} = K_{P2_1}\Delta E_{q_1}'+K_{P2_2}\Delta E_{q_2}'+\Delta K_{P_2} \quad (C.127)$$

$$\Delta Q_{e_1} = K_{Q1_1}\Delta E_{q_1}'+K_{Q1_2}\Delta E_{q_2}'+\Delta K_{Q_1} \quad (C.128)$$

$$\Delta Q_{e_2} = K_{Q2_1}\Delta E_{q_1}'+K_{Q2_2}\Delta E_{q_2}'+\Delta K_{Q_2} \quad (C.129)$$

where K_{P1_1}, K_{P1_2}, K_{P2_1}, K_{P2_2}, K_{Q1_1}, K_{Q1_2}, K_{Q2_1} and K_{Q2_2} are all constants depending on the two generator parameters and initial loading conditions defined as,

$$K_{P1_1} = 2X_{21i}E_{q1o}'+X_{22i}E_{q2o}' \quad (C.130)$$

$$K_{P1_2} = X_{22i}E_{q1o}'+2X_{23i}E_{q2o}' \quad (C.131)$$

$$\Delta K_{P_1} = E_{q1o}'^2\Delta X_{21}+E_{q2o}'^2\Delta X_{23}+E_{q1o}'E_{q2o}'\Delta X_{22} \quad (C.132)$$

$$K_{P2_1} = 2X_{24i}E_{q1o}'+X_{25i}E_{q2o}' \quad (C.133)$$

$$K_{P2_2} = X_{25i}E_{q1o}'+2X_{26i}E_{q2o}' \quad (C.134)$$

$$\Delta K_{P_2} = E_{q1o}'^2\Delta X_{24}+E_{q2o}'^2\Delta X_{26}+E_{q1o}'E_{q2o}'\Delta X_{25} \quad (C.135)$$

$$K_{Q1_1} = 2X_{45i}E_{q1o}'+X_{46i}E_{q2o}' \quad (C.136)$$

$$K_{Q1_2} = X_{46i}E_{q1o}'-2X_{47i}E_{q2o}' \quad (C.137)$$

$$\Delta K_{Q_1} = E_{q1o}'^2\Delta X_{45}-E_{q2o}'^2\Delta X_{47}+E_{q1o}'E_{q2o}'\Delta X_{46} \quad (C.138)$$

$$K_{Q2_1} = 2X_{48i}E_{q1o}'+X_{49i}E_{q2o}' \quad (C.139)$$

$$K_{Q2_2} = X_{49i}E_{q1o}'+2X_{50i}E_{q2o}' \quad (C.140)$$

$$\Delta K_{Q_2} = E_{q1o}'^2\Delta X_{48}+E_{q2o}'^2\Delta X_{50}+E_{q1o}'E_{q2o}'\Delta X_{49} \quad (C.140)$$

X_{1_i} to X_{50_i} are the initial values of the constants X_1 to X_{50} as defined above in Appendix-C.2. The ΔX term will be defined as the change in the value of this constant as the loading conditions move to a new point, after the load disturbance is applied on the generator. For example: $\Delta X_{48} = X_{48} - X_{48i}$.

Linearizing the terminal voltage equation (3.32) we get,

$$\Delta V_t = K_{V_1}\Delta E_{q_1}'+K_{V_2}\Delta E_{q_2}'+\Delta K_V \quad (C.141)$$

where

$$K_{V_1} = (V_{d_0}/V_{t_0})X_{17i}+(V_{q_0}/V_{t_0})X_{19i} \quad (C.142)$$

$$K_{V_2} = (V_{d_0}/V_{t_0})X_{18i}+(V_{q_0}/V_{t_0})X_{20i} \quad (C.143)$$

$$\Delta K_V = (V_{d_0}/V_{t_0})(E_{qo_1}'\Delta X_{17}+E_{qo_2}'\Delta X_{18})+(V_{q_0}/V_{t_0})(E_{qo_1}'\Delta X_{19}+E_{qo_2}'\Delta X_{20}) \quad (C.144)$$

Appendix-D

MATLAB SIMULATION PROGRAMS

D.1 Stand-alone non-linear simulation algorithm:

GEN4

```
function XDOT=GEN4(t,x)
global PEV QEV VTV DELV
```

DATA:

```
P0=0.4; Q0=0.3; VT0=1;W0=1; M=0.82451; D=0; XD=1.518; XDI=0.18; XQ=0.865; TDI0=3.267;
% BASE KVA=1375. P0=0.4=550Kw , Q0=0.3=412.5Kvar >> PF=0.8
KA=400; TA=0.007; KF=0.005; TF=0.6; KE=1; TE=0.08; SE=0.3;
K1=1; K2=1; K3=1; KG=5; KIN=16; KD=0; T1=0.005; T2=0.05; WREF=1;
```

CALCULATE PARAMETERS BEFORE DISTURBANCE:

```
VD0=P0*VT0/sqrt(P0^2+(Q0+VT0^2/XQ)^2);
IQ0=VD0/XQ;
VQ0=sqrt(VT0^2-VD0^2);
ID0=(P0-IQ0*VQ0)/VD0;
EQI0=VQ0+XDI*ID0;
DEL0=asin(VD0/VT0);
EFD0=EQI0+(XD-XDI)*ID0;
VR0=(KE+SE)*EFD0;
VREF=EFD0/KA+VT0;
```

LOAD DISTURBANCE:

```
PL=P0; QL=Q0;
  if t>1,
  PL=0.8;
  QL=0.6;
```

```
%WREF=1.05;
end
```

CALCULATE PARAMETERS AFTER LOAD DISTURBANCE:

```
RL=VT0^2*PL/(PL^2+QL^2); L=VT0^2*QL/(PL^2+QL^2); XL=x(1)*L;
KI=1/(RL^2+(XDI+XL)*(XL+XQ));
KV=sqrt(KI^2*RL^2*XQ^2+(1-KI*XDI*(XL+XQ))^2);
Kq=1+KI*(XD-XDI)*(XL+XQ);
KP=KI^2*RL*XQ*(XQ+XL)+KI*RL-KI^2*RL*XDI*(XL+XQ);
KQ=KI*(XL+XQ)-KI^2*XDI*(XL+XQ)^2-KI^2*RL^2*XQ;
ID=KI*(XQ+XL)*x(2);
IQ=KI*RL*x(2);
VD=XQ*IQ;
VQ=x(2)-XDI*ID;
VT=sqrt(VD^2+VQ^2);
PE=VD*ID+VQ*IQ; QE=VQ*ID-VD*IQ; VTV=[VTV;VT]; PEV=[PEV;PE]; QEV=[QEV;QE];
```

GOVERNOR EQUATION COEFFICIENTS:

```
m1 =KG*KP/M-KP*KD*(2*Kq/TDI0+D/M)/M;
m2 =2*KP*KD/(M*TDI0);
m3 =KG*D/M-KIN-KD*D^2/(M^2);
m4 =KD*D/(M^2)-KG/M+2*KD/(M*T1);
m5 =K1*KD/(M*T2)+2*K1*KD/(M*T1);
m6 =K1*KD*K2*K3/(M*T2);
```

DYNAMIC EQUATIONS:

```
XDOT(1)=(1/M)*(x(8)-PE-D*x(1));
XDOT(2)=(1/TDI0)*(x(3)-Kq*x(2));
XDOT(3)=-(KE+SE)*x(3)/TE+x(4)/TE;
XDOT(4)=-KA*VT/TA-x(4)/TA-KA*x(5)/TA+KA*VREF/TA;
XDOT(5)=-KA*KF*VT/(TF*TA)-KF*x(4)/(TA*TF)-(KA*KF+TA)*x(5)/(TA*TF)+KA*KF*VREF/(TA*TF);
XDOT(6)=-x(6)/T2+K2*K3*x(7)/T2;
XDOT(7)=m1*x(2)^2+m2*x(2)*x(3)+m3*x(1)+m4*x(8)-m5*x(6)+m6*x(7)+KIN*WREF;
XDOT(8)=(K1/T2+2*K1/T1)*x(6)-K1*K2*K3*x(7)/T2-2*x(8)/T1;
XDOT(9)=x(1)-1;
```

DIFFERENTIAL EQUATIONS SOLVING ALGORITHM (SIM4):

```
t0=0; tf=5; global PEV QEV VTV DELV
```

DATA:

```
P0=0.4; Q0=0.3; VT0=1; Wi=1; D=0; M=0.82451;D=0;XD=1.518;XDI=0.18;XQ=0.865;TDI0=3.267;
KA=400;TA=0.007;KE=1;TE=0.08;SE=0.3; K1=1;K2=1;K3=1;
% BASE KVA=1375, P0=0.4=550KW , Q0=0.3=412.5Kvar >> PF=0.8
```

CALCULATE SYSTEM INITIAL OPERATING VALUES:

```
RL0=VT0^2*P0/(P0^2+Q0^2); XL0=VT0^2*Q0/(P0^2+Q0^2);
VD0=P0*VT0/sqrt(P0^2+(Q0+VT0^2/XQ)^2);
IQ0=VD0/XQ;
VQ0=sqrt(VT0^2-VD0^2);
ID0=(P0-IQ0*VQ0)/VD0;
EQI0=VQ0+XDI*ID0;
DEL0=asin(VD0/VT0);
EFD0=EQI0+(XD-XDI)*ID0;
PM0=P0+D*Wi;
VR0=(KE+SE)*EFD0;
VREF=EQI0/KA+VT0;
KI0=1/(RL0^2+(XDI+XL0)*(XL0+XQ));
KV0=sqrt(KI0^2*RL0^2*XQ^2+(1-KI0*XDI*(XL0+XQ))^2);
V30=(-KA*KV0*EQI0-VR0+KA*VREF)/KA;
X70=PM0/(K1*K2*K3);
X60=K2*K3*X70;
```

STATES INITIAL VALUES VECTOR:

```
x0=[Wi EQI0 EFD0 VR0 V30 X60 X70 PM0 DEL0]'; tol=0.01; trace=0;
```

DIFFERENTIAL EQUATIONS NUMERICAL INTEGRATION (ODE4):

```
[t,x]=ODE45('GEN4',t0,tf,x0,tol,trace);
```

PLOT RESULTS

D.2 Stand-alone linear simulation algorithm

G4

DATA:

```
KG=5; KIN=16; KD=0; M=0.84251; XD=1.518; XDI=0.18; XQ=0.865; TDI0=3.267; DMP=0;
P0=0.4; Q0=0.3 ; VT0=1;
KA=400; KF=0.005; KE=1; TA=0.007; TE=0.08; TF=0.6; SE=0.3; K1=1; K2=1; K3=1; T1=0.005; T2=0.05;
% BASE KVA=1375, P0=0.4=550KW , Q0=0.3=412.5Kvar >> PF=0.8
```

CALCULATE PARAMETERS BEFORE DISTURBANCE:

```
VD0=P0*VT0/sqrt(P0^2+(Q0+VT0^2/XQ)^2);
IQ0=VD0/XQ;
VQ0=sqrt(VT0^2-VD0^2);
ID0=(P0-IQ0*VQ0)/VD0;
EQI0=VQ0+XDI*ID0;
DEL0=asin(VD0/VT0);
EFD0=EQI0+(XD-XDI)*ID0;
PM0=P0;
RL0=VT0^2*P0/(P0^2+Q0^2);
XL0=VT0^2*Q0/(P0^2+Q0^2);
KI0=1/(RL0^2+(XQ+XL0)*(XDI+XL0));
KP0=KI0^2*RL0*XQ*(XL0+XQ)+KI0*RL0-KI0^2*RL0*XDI*(XL0+XQ);
KQ0=KI0*(XL0+XQ)-KI0^2*XDI*(XL0+XQ)^2-KI0^2*RL0^2*XQ;
KV0=sqrt(KI0^2*RL0^2*XQ^2+(1-KI0*XDI*(XL0+XQ))^2);
Kq0=1+KI0*(XD-XDI)*(XL0+XQ);
```

LOAD DISTURBANCE:

```
P=0.8; Q=0.3;
```

CALCULATE PARAMETERS AFTER LOAD DISTURBANCE:

```
RL=VT0^2*P/(P^2+Q^2);
XL=VT0^2*Q/(P^2+Q^2);
KI=1/(RL^2+(XQ+XL)*(XDI+XL));
KP=KI^2*RL*XQ*(XL+XQ)+KI*RL-KI^2*RL*XDI*(XL+XQ);
KQ=KI*(XL+XQ)-KI^2*XDI*(XL+XQ)^2-KI^2*RL^2*XQ;
KV=sqrt(KI^2*RL^2*XQ^2+(1-KI*XDI*(XL+XQ))^2);
Kq=1+KI*(XD-XDI)*(XL+XQ);
KPD=KP-KP0; KQD=KQ-KQ0; KVD=KV-KV0; KqD=Kq-Kq0;
```

GOVERNOR EQUATION COEFFICIENTS

```
m1=KG*DMP/M-KIN-DMP^2*KD/(M^2);
m2=(2*K_G*K_Po*EQI0-2*K_D*DMP*K_Po*EQI0)/M+(2*KD*KP0*EFD0-4*K_D*K_Po*Kq_o*EQI0)/(M*TDI0);
m3=2*KD*KP0*EQI0/(M*TDI0);
m4=(KG*EQI0^2-DMP*KD*EQI0^2)/M+(2*KD*EQI0*EFD0-2*KD*Kq0*EQI0^2)/(M*TDI0);
m5=2*KD*EQI0^2*KP0/(M*TDI0);
m6=K1*KD/(M*T2)+2*K1*KD/(M*T1);
m7=K1*K2*K3*KD/(M*T2);
m8=KD*DMP/(M^2)-KG/M+2*KD/(M*T1);
KWD=m4*KPD-m5*KqD;
```

' A ' MATRIX:

```
A11=-DMP/M;                  A12=-2*KP0*EQI0/M;             A18=1/M;
A22=-Kq0/TDI0;               A23=1/TDI0;                    A33=-(KE+SE)/TE;
A34=1/TE;                    A42=-KA*KV0/TA;                A44=-1/TA;
A45=-KA/TA;                  A52=-KA*KV0*KF/(TA*TF);        A54=-KF/(TA*TF);
A55=-(KA*KF+TA)/(TA*TF);     A66=-1/T2;                     A67=K2*K3/T2;
A71=m1;                      A72=m2;                        A73=m3;
A76=-m6;                     A77=m7;                        A78=m8;
A86=K1/T2+2*K1/T1;           A87=-K1*K2*K3/T2;              A88=-2/T1;
```

' B ' MATRIX:

```
DWREF=0.05; DVREF=0.05;
B13=-EQI0^2*KPD/M;   B23=-EQI0*KqD/TDI0;   B42=DVREF*KA/TA; B43=-KA*EQI0*KVD/TA;
B52=DVREF*KA*KF/(TA*TF); B53=-KA*KF*EQI0*KVD/(TA*TF);   B71=DWREF*KIN; B73=KWD;
```

' C ' MATRIX:

```
C1=[1 0 0 0 0 0 0 0 0];  C2=[0 KV0 0 0 0 0 0 0 0];  C3=[0 2*KP0*EQI0 0 0 0 0 0 0 0];  C4=[0 2*KQ0*EQI0 0 0 0 0 0 0 0];
C5=[0 0 0 0 0 0 0 1 0];  C6=[0 1 0 0 0 0 0 0 0];  C7=[0 0 0 0 0 0 0 0 1];
D1=[0 0 0];  D2=[0 0 EQI0*KVD];  D3=[0 0 EQI0^2*KPD];  D4=[0 0 EQI0^2*KQD];  D5=[0 0 0];  D6=[0 0 0];
D7=[0 0 0];  D8=[0 0 0];
```

STEP FUNCTION SIMULATION
PLOT OUTPUT RESULTS

D.3 Two parallel generators non-linear simulation algorithm:

GEN5

function XDOT=GEN5(t,x); global PEV1 QEV1 VTV PEV2 QEV2

DATA:

VT0	=	1;	W0	=	1;	D=0;
M1	=	0.82451;	M2	=	1.051;	
XD1	=	1.518;	XD2	=	1.0741;	
XQ1	=	0.865;	XQ2	=	0.6446;	
XDI1	=	0.18;	XDI2	=	0.1553;	
TDI01	=	3.267;	TDI02	=	3.446;	
KA1	=	400;	KA2	=	400;	
TA1	=	0.007;	TA2	=	0.007;	
KF1	=	0.005;	KF2	=	0.008;	
TF1	=	0.6;	TF2	=	0.5;	
KE1	=	1;	KE2	=	1;	
TE1	=	0.08;	TE2	=	0.1;	
SE1	=	0.3;	SE2	=	0.4;	
K11	=	1;	K12	=	1;	
K21	=	1;	K22	=	1;	
K31	=	1;	K32	=	1;	
T11	=	0.005;	T12	=	0.001;	
T21	=	0.05;	T22	=	0.01;	
KG1	=	5;	KG2	=	5;	
KIN1	=	16;	KIN2	=	16;	
KD1	=	0;	KD2	=	0;	
WREF1=		1;	WREF2=		1;	

% Power factor = 0.8

```
% Total initial load is P0=1000 KW, Q0=750 KVAR, pf=0.8
% P1B=1100KW (1375KVA), P2B=1360KW(1700KVA)
% Initially the 1000KW=0.7272 p.u is shared equally as follows:
P01=0.3252;  P02=0.40207; % Condition for equal real power sharing;
P0=0.7272; Q0=0.5454;
```

CALCULATE PARAMETERS BEFORE LOAD DISTURBANCE:

```
VD0=P0*VT0/sqrt(P0^2+(Q0+(1/XQ1+1/XQ2)*VT0^2)^2);
VQ0=sqrt(VT0^2-VD0^2);
IQ01=VD0/XQ1;                    IQ02=VD0/XQ2;
ID01=(P01-VQ0*IQ01)/VD0;
Q01=VQ0*ID01-VD0*IQ01;           P02=P0-P01; Q02=Q0-Q01;
ID02=(P02-VQ0*IQ02)/VD0;
EQI01=VQ0+XDI1*ID01;             EQI02=VQ0+XDI2*ID02;
EFD01=EQI01+(XD1-XDI1)*ID01;     EFD02=EQI02+(XD2-XDI2)*ID02;
VR01=(KE1+SE1)*EFD01;            VR02=(KE2+SE2)*EFD02;
VREF1=EFD01/KA1+VT0;
VREF2=EFD02/KA2+VT0;
```

LOAD DISTURBANCE:

The resistive load is increased from 1000Kw to 2000Kw in step (0.72 p.u)

Now P_L=2000KW = 1.4545 p.u

```
PL=P0; QL=Q0;
  if t>1,
  PL=1.4545;
  end
```

CALCULATE PARAMETERS & COEFFICIENTS AFTER LOAD DISTURBANCE:

```
RL=VT0^2*PL/(PL^2+QL^2); XL=VT0^2*QL/(PL^2+QL^2);
AX1 =XQ1+XL+XL*XQ1/XQ2;                AX2 =RL+RL*XDI1/XDI2;
AX3 =(RL/XDI2)*(x(10)-x(2));           AX4 =x(2)-(XL/XDI2)*(x(10)-x(2));
AX5 =XDI1+XL+XL*XDI1/XDI2;             AX6 =RL+RL*XQ1/XQ2;
AX7 =AX1/(AX1*AX5+AX2*AX6);            AX8 =AX6/(AX1*AX5+AX2*AX6);
AX9 =AX7+AX7*XL/XDI2+AX8*RL/XDI2;      AX10=AX7*XL/XDI2+AX8*RL/XDI2;
AX11=(1/AX6)*(1+XL/XDI2-AX5*AX9);      AX12=(1/AX6)*(AX5*AX10-XL/XDI2);
AX13=XQ1*AX11/XQ2;                     AX14=XQ1*AX12/XQ2;
AX15=(1/XDI2)*(XDI1*AX9-1);            AX16=(1/XDI2)*(1-XDI1*AX10);
AX17=XQ1*AX11;                         AX18=XQ1*AX12;
AX19=1-XDI1*AX9;                       AX20=XDI1*AX10;
AX21=AX17*AX9+AX19*AX11;
AX22=AX18*AX9-AX17*AX10+AX19*AX12+AX20*AX11;
AX23=AX20*AX12-AX18*AX10;              AX24=AX17*AX15+AX19*AX13;
AX25=AX17*AX16+AX18*AX15+AX19*AX14+AX20*AX13;
AX26=AX18*AX16+AX20*AX14;
AX27=XDI1*AX9-XD1*AX9-1;               AX28=XD1*AX10-XDI1*AX10;
AX29=XDI2*AX15-XD2*AX15;               AX30=XDI2*AX16-XD2*AX16-1;
AX31=2*AX21*AX27/TDI01+AX22*AX29/TDI02;
AX32=(2*AX21*AX28+AX22*AX27)/TDI01+(AX22*AX30+2*AX23*AX29)/TDI02;
AX33=AX22*AX28/TDI01+2*AX23*AX30/TDI02;  AX34=2*AX21/TDI01;
AX35=AX22/TDI01;                       AX36=AX22/TDI02;
AX37=2*AX23/TDI02;                     AX38=2*AX24*AX27/TDI01+AX25*AX29/TDI02;
AX39=(2*AX24*AX28+AX25*AX27)/TDI01+(AX25*AX30+2*AX26*AX29)/TDI02;
AX40=AX25*AX28/TDI01+2*AX26*AX30/TDI02;
AX41=2*AX24/TDI01;                     AX42=AX25/TDI01;
AX43=AX25/TDI02;                       AX44=2*AX26/TDI02;

ID1=AX9*x(2)-AX10*x(10);               ID2=AX15*x(2)+AX16*x(10);
IQ1=AX11*x(2)+AX12*x(10);              IQ2=AX13*x(2)+AX14*x(10);
VD=XQ1*IQ1;                            VQ=x(2)-XDI1*ID1;
VT=sqrt(VD^2+VQ^2);
PE1=VD*ID1+VQ*IQ1;                     QE1=VQ*ID1-VD*IQ1;
PE2=VD*ID2+VQ*IQ2;                     QE2=VQ*ID2-VD*IQ2;
```

```
PDOT1=AX31*x(2)^2+AX32*x(2)*x(10)+AX33*x(10)^2+AX34*x(2)*x(3)+AX35*x(10)*x(3)+
        AX36*x(2)*x(11)+AX37*x(10)*x(11);
PDOT2=AX38*x(2)^2+AX39*x(2)*x(10)+AX40*x(10)^2+AX41*x(2)*x(3)+AX42*x(10)*x(3)+
        AX43*x(2)*x(11)+AX44*x(10)*x(11);

VTV=[VTV;VT]; PEV1=[PEV1;PE1]; QEV1=[QEV1;QE1]; PEV2=[PEV2;PE2]; QEV2=[QEV2;QE2];
```

SPEED CONTROLLERS EQUATIONS COEFFICIENTS:

```
m11 = KG1*D/M1-KIN1-KD1*D^2/M1^2;
m21 = KG1/M1-KD1*D/M1^2;
m31 = 2*KD1/(M1*T11)-KG1/M1+KD1*D/M1^2;
m41 = KD1/M1;
m51 = (KD1/M1)*(K11/T21+2*K11/T11);
m61 = K11*KD1*K21*K31/(M1*T21);

m12 = KG2*D/M2-KIN2-KD2*D^2/M2^2;
m22 = KG2/M2-KD2*D/M2^2;
m32 = 2*KD2/(M2*T12)-KG2/M2+KD2*D/M2^2;
m42 = KD2/M2;
m52 = (KD2/M2)*(K12/T22+2*K12/T12);
m62 = K12*KD2*K22*K32/(M2*T22);
```

DIFFERENTIAL EQUATIONS:

```
XDOT(1)  = (1/M1)*(x(8)-PE1-D*x(1));
XDOT(2)  = (1/TDI01)*(x(3)-(XD1-XDI1)*ID1);
XDOT(3)  = -(KE1+SE1)*x(3)/TE1+x(4)/TE1;
XDOT(4)  = -KA1*VT/TA1-x(4)/TA1-KA1*x(5)/TA1+KA1*VREF1/TA1;
XDOT(5)  = -KA1*KF1*VT/(TF1*TA1)-KF1*x(4)/(TA1*TF1)-(KA1*KF1+TA1)* x(5)/(TA1*TF1)+
            KA1*KF1*VREF1/(TA1*TF1);
XDOT(6)  = -x(6)/T21+K21*K31*x(7)/T21;
XDOT(7)  = m11*x(1)+m21*PE1+m31*x(8)+m41*PDOT1-m51*x(6)+m61*x(7)+KIN1*WREF1;
XDOT(8)  = (K11/T21+2*K11/T11)*x(6)-K11*K21*K31*x(7)/T21-2*x(8)/T11;
XDOT(9)  = (1/M2)*(x(16)-PE2-D*x(9));
XDOT(10) = (1/TDI02)*(x(11)-(XD2-XDI2)*ID2);
XDOT(11) = -(KE2+SE2)*x(11)/TE2+x(12)/TE2;
XDOT(12) = -KA2*VT/TA2-x(12)/TA2-KA2*x(13)/TA2+KA2*VREF2/TA2;
XDOT(13) = -KA2*KF2*VT/(TF2*TA2)-KF2*x(12)/(TA2*TF2)-(KA2*KF2+TA2)*x(13)/(TA2*TF2)
            +KA2*KF2*VREF2/(TA2*TF2);
XDOT(14) = -x(14)/T22+K22*K32*x(15)/T22;
XDOT(15) = m12*x(9)+m22*PE2+m32*x(16)+m42*PDOT2-m52*x(14)+m62*x(15)+KIN2*WREF2;
XDOT(16) = (K12/T22+2*K12/T12)*x(14)-K12*K22*K32*x(15)/T22-2*x(16)/T12;
XDOT(17) = (1/MS)*(x(8)+x(16)-PE1-PE2-D*x(17));
XDOT(18) = x(1)-x(9);
XDOT(19) = x(1)-1;
XDOT(20) = x(9)-1;
```

DIFFERENTIAL EQUATIONS SOLVING ALGORITHM (SIM5):

```
t0=0; tf=6; global PEV1 QEV1 VTV PEV2 QEV2
```

CALCULATE INITIAL OEPRATING VALUES:

```
P0=0.7272; Q0=0.5454;
VT0=1; Wi=1; D=0; P01=0.3252; P02=0.40207;
RL0=VT0^2*P0/(P0^2+Q0^2); XL0=VT0^2*Q0/(P0^2+Q0^2);
VD0=P0*VT0/sqrt(P0^2+(Q0+(1/XQ1+1/XQ2)*VT0^2)^2);
VQ0=sqrt(VT0^2-VD0^2);
IQ01=VD0/XQ1;                   IQ02=VD0/XQ2;
ID01=(P01-VQ0*IQ01)/VD0;        ID02=(P02-VQ0*IQ02)/VD0;
Q01=VQ0*ID01-VD0*IQ01;          Q02=Q0-Q01;
EQI01=VQ0+XDI1*ID01;            EQI02=VQ0+XDI2*ID02;
EFD01=EQI01+(XD1-XDI1)*ID01;    EFD02=EQI02+(XD2-XDI2)*ID02;
VR01=(KE1+SE1)*EFD01;           VR02=(KE2+SE2)*EFD02;
```

```
VREF1=EFD01/KA1+VT0;              VREF2=EFD02/KA2+VT0;
PM01=P01+D*Wi;                    PM02=P02+D*Wi;
VF01=(-KA1*VT0-VR01+KA1*VREF1)/KA1;  VF02=(-KA2*VT0-VR02+KA2*VREF2)/KA2;
X701=PM01/(K11*K21*K31);          X702=PM02/(K12*K22*K32);
X601=K21*K31*X701;                X602=K22*K32*X702;
DEL10=asin(VD0/VT0);              DEL20=DEL10; DEL120=0;
```

INITIAL STAES VECTOR:

```
x0=[Wi EQI01 EFD01 VR01 VF01 X601 X701 PM01 Wi EQI02 EFD02 VR02 VF02 X602 X702 PM02
    Wi DEL120 DEL10 DEL20 0]';

tol=0.001; trace=0;
[t,x]=ODE45('GEN5',t0,tf,x0,tol,trace); PLOT RESULTS
```

D.4 Two parallel generators linear simulation Algorithm:

G5

```
VT0=1;  W0=1;  DMP=0;
% Power factor = 0.8
% Total initial load is P0=1000 KW, Q0=750 KVAR, pf=0.8
% P1B=1100KW (1375KVA), P2B=1360KW(1700KVA)
% Initially the 1000KW=0.7272 p.u is shared equally
%-------------------------------------------------------------------
VD0=P0*VT0/sqrt(P0^2+(Q0+(1/XQ1+1/XQ2)*VT0^2)^2);
VQ0=sqrt(VT0^2-VD0^2);
IQ01=VD0/XQ1;                     IQ02=VD0/XQ2;
ID01=(P01-VQ0*IQ01)/VD0;
Q01=VQ0*ID01-VD0*IQ01;            Q02=Q0-Q01;
ID02=(P02-VQ0*IQ02)/VD0;
Q0=Q01+Q02;
PM01=P01;                         PM02=P02;
EQI01=VQ0+XDI1*ID01;              EQI02=VQ0+XDI2*ID02;
EFD01=EQI01+(XD1-XDI1)*ID01;      EFD02=EQI02+(XD2-XDI2)*ID02;
VR01=(KE1+SE1)*EFD01;             VR02=(KE2+SE2)*EFD02;
VREF1=EFD01/KA1+VT0;              VREF2=EFD02/KA2+VT0;
VF01=(-KA1*VT0-VR01+KA1*VREF1)/KA1;  VF02=(-KA2*VT0-VR02+KA2*VREF2)/KA2;
XG01=PM01/(K11*K21*K31);          XG02=PM02/(K12*K22*K32);
XA01=K21*K31*XG01;                XA02=K22*K32*XG02;
DEL10=asin(VD0/VT0);              DEL20=DEL10; DEL120=0;
DEL0=asin(VD0/VT0);
%------------------------------------------------------------------------------------------------
RLi=VT0^2*P0/(P0^2+Q0^2);         XLi =VT0^2*Q0/(P0^2+Q0^2);
X1i =XQ1+XLi+XLi*XQ1/XQ2;         X2i =RLi+RLi*XDI1/XDI2;
X5i =XDI1+XLi+XLi*XDI1/XDI2;      X6i =RLi+RLi*XQ1/XQ2;
X7i =X1i/(X1i*X5i+X2i*X6i);       X8i =X6i/(X1i*X5i+X2i*X6i);
X9i =X7i+X7i*XLi/XDI2+X8i*RLi/XDI2;  X10i=X7i*XLi/XDI2+X8i*RLi/XDI2;
X11i=(1/X6i)*(1+XLi/XDI2-X5i*X9i);   X12i=(1/X6i)*(X5i*X10i-XLi/XDI2);
X13i=XQ1*X11i/XQ2;                X14i=XQ1*X12i/XQ2;
X15i=(1/XDI2)*(XDI1*X9i-1);       X16i=(1/XDI2)*(1-XDI1*X10i);
X17i=XQ1*X11i;                    X18i=XQ1*X12i;
X19i=1-XDI1*X9i;                  X20i=XDI1*X10i;
X21i=X17i*X9i+X19i*X11i;
X22i=X18i*X9i-X17i*X10i+X19i*X12i+X20i*X11i;
X23i=X20i*X12i-X18i*X10i;         X24i=X17i*X15i+X19i*X13i;
X25i=X17i*X16i+X18i*X15i+X19i*X14i+X20i*X13i;
X26i=X18i*X16i+X20i*X14i;
X27i=XDI1*X9i-XD1*X9i-1;          X28i=XD1*X10i-XDI1*X10i;
X29i=XDI2*X15i-XD2*X15i;          X30i=XDI2*X16i-XD2*X16i-1;
```

```
X31i=2*X21i*X27i/TDI01+X22i*X29i/TDI02;
X32i=(2*X21i*X28i+X22i*X27i)/TDI01+(X22i*X30i+2*X23i*X29i)/TDI02;
X33i=X22i*X28i/TDI01+2*X23i*X30i/TDI02; X34i=2*X21i/TDI01;
X35i=X22i/TDI01;                  X36i=X22i/TDI02;
X37i=2*X23i/TDI02;                X38i=2*X24i*X27i/TDI01+X25i*X29i/TDI02;
X39i=(2*X24i*X28i+X25i*X27i)/TDI01+(X25i*X30i+2*X26i*X29i)/TDI02;
X40i=X25i*X28i/TDI01+2*X26i*X30i/TDI02;
X41i=2*X24i/TDI01;              X42i=X25i/TDI01;
X43i=X25i/TDI02;                X44i=2*X26i/TDI02;
X45i=X9i*X19i-X11i*X17i;          X46i=X9i*X20i-X10i*X19i-X11i*X18i-X12i*X17i;
X47i=X10i*X20i+X12i*X18i;         X48i=X15i*X19i-X13i*X17i;
X49i=X16i*X19i+X15i*X20i-X14i*X17i-X13i*X18i; X50i=X16i*X20i-X14i*X18i;
%-------------------------------------------------------------------------------------------------
P1=1.4545; % Total active load is increased from 1000 KW to 2000KW (0.73 p.u);
Q1=Q0;

RL=VT0^2*P1/(P1^2+Q1^2);                XL=VT0^2*Q1/(P1^2+Q1^2);
X1 =XQ1+XL+XL*XQ1/XQ2;                  X2 =RL+RL*XDI1/XDI2;
X5 =XDI1+XL+XL*XDI1/XDI2;               X6 =RL+RL*XQ1/XQ2;
X7 =X1/(X1*X5+X2*X6);                   X8 =X6/(X1*X5+X2*X6);
X9 =X7+X7*XL/XDI2+X8*RL/XDI2;           X10=X7*XL/XDI2+X8*RL/XDI2;
X11=(1/X6)*(1+XL/XDI2-X5*X9);           X12=(1/X6)*(X5*X10-XL/XDI2);
X13=XQ1*X11/XQ2;                        X14=XQ1*X12/XQ2;
X15=(1/XDI2)*(XDI1*X9-1);               X16=(1/XDI2)*(1-XDI1*X10);
X17=XQ1*X11;                            X18=XQ1*X12;
X19=1-XDI1*X9;                          X20=XDI1*X10;
X21=X17*X9+X19*X11;
X22=X18*X9-X17*X10+X19*X12+X20*X11;
X23=X20*X12-X18*X10;                    X24=X17*X15+X19*X13;
X25=X17*X16+X18*X15+X19*X14+X20*X13;
X26=X18*X16+X20*X14;
X27=XDI1*X9-XD1*X9-1;                   X28=XD1*X10-XDI1*X10;
X29=XDI2*X15-XD2*X15;                   X30=XDI2*X16-XD2*X16-1;
X31=2*X21*X27/TDI01+X22*X29/TDI02;
X32=(2*X21*X28+X22*X27)/TDI01+(X22*X30+2*X23*X29)/TDI02;
X33=X22*X28/TDI01+2*X23*X30/TDI02;      X34=2*X21/TDI01;
X35=X22/TDI01;                          X36=X22/TDI02;
X37=2*X23/TDI02;                        X38=2*X24*X27/TDI01+X25*X29/TDI02;
X39=(2*X24*X28+X25*X27)/TDI01+(X25*X30+2*X26*X29)/TDI02;
X40=X25*X28/TDI01+2*X26*X30/TDI02;
X41=2*X24/TDI01;                        X42=X25/TDI01;
X43=X25/TDI02;                          X44=2*X26/TDI02;
X45=X9*X19-X11*X17;                     X46=X9*X20-X10*X19-X11*X18-X12*X17;
X47=X10*X20+X12*X18;                    X48=X15*X19-X13*X17;
X49=X16*X19+X15*X20-X14*X17-X13*X18;    X50=X16*X20-X14*X18;
%--------------------------------------------------------------------
KP11=2*X21i*EQI01+X22i*EQI02;           KP12=X22i*EQI01+2*X23i*EQI02;
KP21=2*EQI01*X24i+X25i*EQI02;           KP22=X25i*EQI01+2*EQI02*X26i;
KP1D=EQI01^2*(X21-X21i)+EQI01*EQI02*(X22-X22i)+EQI02^2*(X23-X23i);
KP2D=EQI01^2*(X24-X24i)+EQI01*EQI02*(X25-X25i)+EQI02^2*(X26-X26i);
Kq1D=EQI01*(X27-X27i)+EQI02*(X28-X28i); Kq2D=EQI01*(X29-X29i)+EQI02*(X30-X30i);
KQ11=2*X45i*EQI01+X46i*EQI02;           KQ12=X46i*EQI01-2*EQI02*X47i;
KQ21=2*EQI01*X48i+X49i*EQI02;           KQ22=X49i*EQI01+2*EQI02*X50i;
KQ1D=EQI01^2*(X45-X45i)+EQI01*EQI02*(X46-X46i)-EQI02^2*(X47-X47i);
KQ2D=EQI01^2*(X48-X48i)+EQI01*EQI02*(X49-X49i)+EQI02^2*(X50-X50i);
KV1=VD0*X17i/VT0+VQ0*X19i/VT0;          KV2=VD0*X18i/VT0+VQ0*X20i/VT0;
KVD=VD0*EQI01*(X17-X17i)/VT0+VD0*EQI02*(X18-X18i)/VT0+VQ0*EQI01*(X19-X19i)/VT0+VQ0*EQI02*(X20-
X20i)/VT0;

m11 = KG1*DMP/M1-KIN1-KD1*DMP^2/M1^2;
m21 = KG1*KP11/M1-KD1*DMP*KP11/M1^2+KD1*KP11*X27i/(M1*TDI01)+KD1*KP12*X29i/(M1*TDI02);
m31 = KP12*KG1/M1-KD1*DMP*KP12/M1^2+KD1*KP11*X28i/(M1*TDI01)+KD1*KP12*X30i/(M1*TDI02);
```

```
m41 = KD1*KP11/(M1*TDI01);
m51 = KD1*KP12/(M1*TDI02);
m61 = (KD1/M1)*(K11/T21+2*K11/T11);
m71 = K11*KD1*K21*K31/(M1*T21);
m81 = 2*KD1/(M1*T21)+KD1*DMP/M1^2-KG1/M1;
KW1D= (KD1/M1)*(-DMP*KP1D/M1+KP11*Kq1D/TDI01+KP12*Kq2D/TDI02)+KG1*KP1D/M1;

m12 = KG2*DMP/M2-KIN2-KD2*DMP^2/M2^2;
m22 = KG2*KP21/M2-KD2*DMP*KP21/M2^2+KD2*KP21*X27i/(M2*TDI01)+KD2*KP22*X29i/(M2*TDI02);
m32 = KP22*KG2/M2-KD2*DMP*KP22/M2^2+KD2*KP21*X28i/(M2*TDI01)+KD2*KP22*X30i/(M2*TDI02);
m42 = KD2*KP21/(M2*TDI01);
m52 = KD2*KP22/(M2*TDI02);
m62 = (KD2/M2)*(K12/T22+2*K12/T12);
m72 = K12*KD2*K22*K32/(M2*T22);
m82 = 2*KD2/(M2*T22)+KD2*DMP/M2^2-KG2/M2;
KW2D= KG2*KP2D/M2+KD2*KP21*Kq1D/(M2*TDI01)+KD2*KP22*Kq2D/(M2*TDI02)-KD2*KP2D*DMP/M2^2;
%------------------------------------------------------------------------------
A11=-DMP/M1; A12=-KP11/M1; A18=1/M1; A110=-KP12/M1;
A22=X27i/TDI01; A23=1/TDI01; A210=X28i/TDI01;
A33=-(KE1+SE1)/TE1; A34=1/TE1;
A42=-KA1*KV1/TA1; A44=-1/TA1; A45=-KA1/TA1; A410=-KA1*KV2/TA1;
A52=-KA1*KF1*KV1/(TA1*TF1); A54=-KF1/(TA1*TF1); A55=-(KA1*KF1+TA1)/(TA1*TF1); A510=-
KA1*KF1*KV2/(TA1*TF1);
A66=-1/T21; A67=K21*K31/T21;
A71=m11; A72=m21; A73=m41; A76=-m61; A77=m71; A78=m81; A710=m31; A711=m51;
A86=K11/T21+2*K11/T11; A87=-K11*K21*K31/T21; A88=-2/T11;
A92=-KP21/M2; A99=-DMP/M2; A910=-KP22/M2; A916=1/M2;
A102=X29i/TDI02; A1010=X30i/TDI02; A1011=1/TDI02;
A1111=-(KE2+SE2)/TE2; A1112=1/TE2;
A122=-KA2*KV1/TA2; A1210=-KA2*KV2/TA2; A1212=-1/TA2; A1213=-KA2/TA2;
A132=-KA2*KF2*KV1/(TA2*TF2); A1310=-KA2*KF2*KV2/(TA2*TF2); A1312=-KF2/(TA2*TF2); A1313=-
(KA2*KF2+TA2)/(TA2*TF2);
A1414=-1/T22; A1415=K22*K32/T22;
A159=m12; A152=m22; A153=m42; A1510=m32; A1511=m52; A1514=-m62; A1515=m72; A1516=m82;
A1614=K12/T22+2*K12/T12; A1615=-K12*K22*K32/T22; A1616=-2/T12;

MATRIX    A

B15=-KP1D/M1;              B95=-KP2D/M2;           B25=Kq1D/TDI01;      B105=Kq2D/TDI02;
B42=0.05*KA1/TA1;          B45=-KA1*KVD/TA1;        B124=0.05*KA2/TA2;      B125=-KA2*KVD/TA2;
B52=0.05*KA1*KF1/(TA1*TF1); B55=-KA1*KF1*KVD/(TA1*TF1);  B134=0.05*KA2*KF2/(TA2*TF2);
B135=-KA2*KF2*KVD/(TA2*TF2);  B71=0.05*KIN1;          B75=KW1D;
B153=0.05*KIN2;B155=KW2D;

MATRIX    B

C1 =[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];              % G-1 freq
C2 =[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0];              % G-2 freq
C3 =[0 KV1 0 0 0 0 0 0 0 KV2 0 0 0 0 0 0 0 0];          % Terminal voltage
C4 =[0 KP11 0 0 0 0 0 0 0 KP12 0 0 0 0 0 0 0 0];        % G-1 Pe
C5 =[0 KP21 0 0 0 0 0 0 0 KP22 0 0 0 0 0 0 0 0];        % G-2 Pe
C6 =[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0];              % G-1 Pm
C7 =[0 KQ11 0 0 0 0 0 0 0 KQ12 0 0 0 0 0 0 0 0];        % G-1 Qe
C8 =[0 KQ21 0 0 0 0 0 0 0 KQ22 0 0 0 0 0 0 0 0];        % G-2 Qe
C9 =[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];              % G-1 Efd
C10=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0];              % G-2 Pm
C11=[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0];              % G-2 Efd
C12=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0];              % G-1 del
C13=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1];              % G-2 del
C14=[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];              % G-1 Eq'1
C15=[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0];              % G-2 Eq'2
%------------------------------------------------
```

```
TF=5;
T1=[1:0.005:TF]; D0=[0 0 0 0 0]; D3=[0 0 0 0 KVD]; D4=[0 0 0 0 KP1D]; D5=[0 0 0 0 KP2D]; D7=[0 0 0 0
KQ1D]; D8=[0 0 0 0 KQ2D];
C=[C1;C2;C3;C4;C5;C7;C8]; D=[D0;D0;D3;D4;D5;D7;D8];
%--------------------------------------------------------------------------------
PLOT RESULTS
```

D.5 Load sharing simulation algorithm:

```
function XDOT=GEN5LS(t,x)
global PEV1 QEV1 VTV PEV2 QEV2

% Total initial load is P0=1600 KW. Q0=1200 KVAR, pf=0.8
% P1B=1100KW (1375KVA), P2B=1100KW(1375KVA)
% Initially the 1600KW=1.163636 p.u is shared unequally as follows:
%                        P01=600KW & P02=1000KW
%
P0=1.163636; P01=0.436363; P02=0.73; Q0=0.87;
% Conditions for equal load sharing must be P01=0.5818(800Kw) & P02=0.5818(800Kw)
%                        Q01=0.4363(600Kvar) & Q02=0.4363(600Kvar)

 M1    =  0.82451;        M2     =  0.82451;
 XD1   =  1.518;          XD2    =  1.518;
 XQ1   =  0.865;          XQ2    =  0.865;
 XDI1  =  0.18;           XDI2   =  0.18;
 TDI01 =  3.267;          TDI02  =  3.267;
 KA1   =  400;            KA2    =  400;
 TA1   =  0.007;          TA2    =  0.007;
 KF1   =  0.005;          KF2    =  0.005;
 TF1   =  0.6;            TF2    =  0.6;
 KE1   =  1;              KE2    =  1;
 TE1   =  0.08;           TE2    =  0.08;
 SE1   =  0.3;            SE2    =  0.3;
 K11   =  1;              K12    =  1;
 K21   =  1;              K22    =  1;
 K31   =  1;              K32    =  1;
 T11   =  0.005;          T12    =  0.005;
 T21   =  0.05;           T22    =  0.05;
 KG1   =  5;              KG2    =  5;
 KIN1  =  16;             KIN2   =  16;
 KD1   =  0;              KD2    =  0;
 WREF1 =  1;              WREF2  =  1;

VD0=P0*VT0/sqrt(P0^2+(Q0+(1/XQ1+1/XQ2)*VT0^2)^2);
VQ0=sqrt(VT0^2-VD0^2);
IQ01=VD0/XQ1;                        IQ02=VD0/XQ2;
ID01=(P01-VQ0*IQ01)/VD0;
Q01=VQ0*ID01-VD0*IQ01;               P02=P0-P01; Q02=Q0-Q01;
ID02=(P02-VQ0*IQ02)/VD0;
EQI01=VQ0+XDI1*ID01;                 EQI02=VQ0+XDI2*ID02;
EFD01=EQI01+(XD1-XDI1)*ID01;         EFD02=EQI02+(XD2-XDI2)*ID02;
VR01=(KE1+SE1)*EFD01;                VR02=(KE2+SE2)*EFD02;
VREF1=EFD01/KA1+VT0;
VREF2=EFD02/KA2+VT0;

Q0=Q01+Q02;
PL=P0; QL=Q0;
RL=VT0^2*PL/(PL^2+QL^2);             XL=VT0^2*QL/(PL^2+QL^2);
```

```
AX1 =XQ1+XL+XL*XQ1/XQ2;            AX2 =RL+RL*XDI1/XDI2;
AX3 =(RL/XDI2)*(x(10)-x(2));       AX4 =x(2)-(XL/XDI2)*(x(10)-x(2));
AX5 =XDI1+XL+XL*XDI1/XDI2;         AX6 =RL+RL*XQ1/XQ2;
AX7 =AX1/(AX1*AX5+AX2*AX6);        AX8 =AX6/(AX1*AX5+AX2*AX6);
AX9 =AX7+AX7*XL/XDI2+AX8*RL/XDI2;  AX10=AX7*XL/XDI2+AX8*RL/XDI2;
AX11=(1/AX6)*(1+XL/XDI2-AX5*AX9);  AX12=(1/AX6)*(AX5*AX10-XL/XDI2);
AX13=XQ1*AX11/XQ2;                 AX14=XQ1*AX12/XQ2;
AX15=(1/XDI2)*(XDI1*AX9-1);        AX16=(1/XDI2)*(1-XDI1*AX10);
AX17=XQ1*AX11;                     AX18=XQ1*AX12;
AX19=1-XDI1*AX9;                   AX20=XDI1*AX10;
AX21=AX17*AX9+AX19*AX11;
AX22=AX18*AX9-AX17*AX10+AX19*AX12+AX20*AX11;
AX23=AX20*AX12-AX18*AX10;          AX24=AX17*AX15+AX19*AX13;
AX25=AX17*AX16+AX18*AX15+AX19*AX14+AX20*AX13;
AX26=AX18*AX16+AX20*AX14;
AX27=XDI1*AX9-XD1*AX9-1;           AX28=XD1*AX10-XDI1*AX10;
AX29=XDI2*AX15-XD2*AX15;           AX30=XDI2*AX16-XD2*AX16-1;
AX31=2*AX21*AX27/TDI01+AX22*AX29/TDI02;
AX32=(2*AX21*AX28+AX22*AX27)/TDI01+(AX22*AX30+2*AX23*AX29)/TDI02;
AX33=AX22*AX28/TDI01+2*AX23*AX30/TDI02;  AX34=2*AX21/TDI01;
AX35=AX22/TDI01;                   AX36=AX22/TDI02;
AX37=2*AX23/TDI02;                 AX38=2*AX24*AX27/TDI01+AX25*AX29/TDI02;
AX39=(2*AX24*AX28+AX25*AX27)/TDI01+(AX25*AX30+2*AX26*AX29)/TDI02;
AX40=AX25*AX28/TDI01+2*AX26*AX30/TDI02;
AX41=2*AX24/TDI01;                 AX42=AX25/TDI01;
AX43=AX25/TDI02;                   AX44=2*AX26/TDI02;

ID1=AX9*x(2)-AX10*x(10);           ID2=AX15*x(2)+AX16*x(10);
IQ1=AX11*x(2)+AX12*x(10);          IQ2=AX13*x(2)+AX14*x(10);
VD=XQ1*IQ1;                        VQ=x(2)-XDI1*ID1;
VT=sqrt(VD^2+VQ^2);

DPLS1=0; DPLS2=0; DQLS1=0; DQLS2=0;

PE1=VD*ID1+VQ*IQ1;       QE1=VQ*ID1-VD*IQ1;
PE2=VD*ID2+VQ*IQ2;       QE2=VQ*ID2-VD*IQ2;

if t>4,
DPLS1=0.5*(PE2-PE1);     DPLS2=0.5*(PE1-PE2);
DQLS1=0.5*(QE2-QE1);     DQLS2=0.5*(QE1-QE2);
end

PE1=PE1+DPLS1;           QE1=QE1+DQLS1;
PE2=PE2+DPLS2;           QE2=QE2+DQLS2;

PDOT1=AX31*x(2)^2+AX32*x(2)*x(10)+AX33*x(10)^2+AX34*x(2)*x(3)+AX35*x(10)*x(3)+AX36*x(2)*x(11)+AX3
7*x(10)*x(11);
PDOT2=AX38*x(2)^2+AX39*x(2)*x(10)+AX40*x(10)^2+AX41*x(2)*x(3)+AX42*x(10)*x(3)+AX43*x(2)*x(11)+AX4
4*x(10)*x(11);

VTV=[VTV;VT];
PEV1=[PEV1;PE1]; QEV1=[QEV1;QE1]; PEV2=[PEV2;PE2]; QEV2=[QEV2;QE2];

m11=KG1*D/M1-KIN1-KD1*D^2/M1^2;
m21=KD1*D/M1^2-KG1/M1+2*KD1/(M1*T11);
m31=KD1/M1;
m41=KG1/M1-KD1*D/M1^2;
m51=KD1*(K11/T21+2*K11/T11)/M1;
m61=KD1*K11*K21*K31/(M1*T21);
m71=KIN1*WREF1;
```

```
m12=KG2*D/M2-KIN2-KD2*D^2/M2^2;
m22=KD2*D/M2^2-KG2/M2+2*KD2/(M2*T12);
m32=KD2/M2;
m42=KG2/M2-KD2*D/M2^2;
m52=KD2*(K12/T22+2*K12/T12)/M2;
m62=KD2*K12*K22*K32/(M2*T22);
m72=KIN2*WREF2;

XDOT(1)  = (1/M1)*(x(8)-PE1-D*x(1));
XDOT(2)  = (1/TDI01)*(x(3)-(XD1-XDI1)*ID1);
XDOT(3)  = -(KE1+SE1)*x(3)/TE1+x(4)/TE1;
XDOT(4)  = KA1*VREF1/TA1-KA1*x(5)/TA1-KA1*VT/TA1-x(4)/TA1;
XDOT(5)  = KA1*KF1*VREF1/(TA1*TF1)-(KA1*KF1+TA1)*x(5)/(TA1*TF1)-KA1*KF1*VT/(TA1*TF1)
           - KF1*x(4)/(TA1*TF1);
XDOT(6)  = -x(6)/T21+K21*K31*x(7)/T21;
XDOT(7)  = m11*x(1)+m21*x(8)+m31*PDOT1+m41*PE1-m51*x(6)+m61*x(7)+m71;
XDOT(8)  = (K11/T21+2*K11/T11)*x(6)-K11*K21*K31*x(7)/T21-2*x(8)/T11;
XDOT(9)  = (1/M2)*(x(16)-PE2-D*x(9));
XDOT(10) = (1/TDI02)*(x(11)-(XD2-XDI2)*ID2);
XDOT(11) = -(KE2+SE2)*x(11)/TE2+x(12)/TE2;
XDOT(12) = KA2*VREF2/TA2-KA2*x(13)/TA2-KA2*VT/TA2-x(12)/TA2;
XDOT(13) = KA2*KF2*VREF2/(TA2*TF2)-(KA2*KF2+TA2)*x(13)/(TA2*TF2)-KA2*KF2*VT/(TA2*TF2)
           - KF2*x(12)/(TA2*TF2);
XDOT(14) = -x(14)/T22+K22*K32*x(15)/T22;
XDOT(15) = m12*x(9)+m22*x(16)+m32*PDOT2+m42*PE2-m52*x(14)+m62*x(15)+m72;
XDOT(16) = (K12/T22+2*K12/T12)*x(14)-K12*K22*K32*x(15)/T22-2*x(16)/T12;
XDOT(17) = x(1)-1;
XDOT(18) = x(9)-1;

t0=0; tf=10;
global PEV1 QEV1 VTV PEV2 QEV2

VT0=1;  Wi=1;  DMP=0;
EQI01=VQ0+XDI1*ID01;              EQI02=VQ0+XDI2*ID02;
EFD01=EQI01+(XD1-XDI1)*ID01;      EFD02=EQI02+(XD2-XDI2)*ID02;
VR01=(KE1+SE1)*EFD01;             VR02=(KE2+SE2)*EFD02;
VREF1=EFD01/KA1+VT0;              VREF2=EFD02/KA2+VT0;
PM01=P01+D*Wi;                    PM02=P02+D*Wi;
VF01=(-KA1*VT0-VR01+KA1*VREF1)/KA1;  VF02=(-KA2*VT0-VR02+KA2*VREF2)/KA2;
XG01=PM01/(K11*K21*K31);          XG02=PM02/(K12*K22*K32);
XA01=K21*K31*XG01;                XA02=K22*K32*XG02;
DEL10=asin(VD0/VT0);              DEL20=DEL10; DEL120=0;

x0=[Wi EQI01 EFD01 VR01 VF01 XA01 XG01 PM01 Wi EQI02 EFD02 VR02 VF02 XA02 XG02 PM02 DEL10 DEL20
Wi]';

tol=0.001; trace=0;

[t,x]=ODE45('GEN5LS',t0,tf,x0,tol,trace);
```

PLOT RESULTS

D.6 Two parallel generators linear simulation algorithm with LQR methods:

G4G5a

% GENERATOR # 1 SIMULATION))

```
VD0=P0*VT0/sqrt(P0^2+(Q0+VT0^2/XQ)^2); IQ0=VD0/XQ;
VQ0=sqrt(VT0^2-VD0^2); ID0=(P0-IQ0*VQ0)/VD0;
EQI0=VQ0+XDI*ID0;
DEL0=asin(VD0/VT0);
EFD0=EQI0+(XD-XDI)*ID0;
PM0=P0;
RL0=VT0^2*P0/(P0^2+Q0^2); XL0=VT0^2*Q0/(P0^2+Q0^2);
KI0=1/(RL0^2+(XQ+XL0)*(XDI+XL0));
KP0=KI0^2*RL0*XQ*(XL0+XQ)+KI0*RL0-KI0^2*RL0*XDI*(XL0+XQ);
KQ0=KI0*(XL0+XQ)-KI0^2*XDI*(XL0+XQ)^2-KI0^2*RL0^2*XQ;
KV0=sqrt(KI0^2*RL0^2*XQ^2+(1-KI0*XDI*(XL0+XQ))^2);
Kq0=1+KI0*(XD-XDI)*(XL0+XQ);

P=0.8; Q=0.2334;
RL=VT0^2*P/(P^2+Q^2); XL=VT0^2*Q/(P^2+Q^2);
KI=1/(RL^2+(XQ+XL)*(XDI+XL));
KP=KI^2*RL*XQ*(XL+XQ)+KI*RL-KI^2*RL*XDI*(XL+XQ);
KQ=KI*(XL+XQ)-KI^2*XDI*(XL+XQ)^2-KI^2*RL^2*XQ;
KV=sqrt(KI^2*RL^2*XQ^2+(1-KI*XDI*(XL+XQ))^2);
Kq=1+KI*(XD-XDI)*(XL+XQ);
KPD=KP-KP0; KQD=KQ-KQ0; KVD=KV-KV0; KqD=Kq-Kq0;

m1=KG*DMP/M-KIN-DMP^2*KD/(M^2);
m2=(2*KG*KP0*EQI0-2*KD*DMP*KP0*EQI0)/M+(2*KD*KP0*EFD0-4*KD*KP0*Kq0*EQI0)/(M*TDI0);
m3=2*KD*KP0*EQI0/(M*TDI0);
m4=(KG*EQI0^2-DMP*KD*EQI0^2)/M+(2*KD*EQI0*EFD0-2*KD*Kq0*EQI0^2)/(M*TDI0);
m5=2*KD*EQI0^2*KP0/(M*TDI0);
m6=K1*KD/(M*T2)+2*K1*KD/(M*T1);
m7=K1*K2*K3*KD/(M*T2);
m8=KD*DMP/(M^2)-KG/M+2*KD/(M*T1);
KWD=m4*KPD-m5*KqD;

A11=-DMP/M;          A12=-2*KP0*EQI0/M;       A18=1/M;
A22=-Kq0/TDI0;       A23=1/TDI0;              A33=-(KE+SE)/TE;
A34=1/TE;            A42=-KA*KV0/TA;          A44=-1/TA;
A45=-KA/TA;          A52=-KA*KV0*KF/(TA*TF);  A54=-KF/(TA*TF);
A55=-(KA*KF+TA)/(TA*TF); A66=-1/T2;           A67=K2*K3/T2;
A71=m1;              A72=m2;                  A73=m3;
A76=-m6;             A77=m7;                  A78=m8;
A86=K1/T2+2*K1/T1;   A87=-K1*K2*K3/T2;        A88=-2/T1;

MATRIX   A1

DWREF=0.05; DVREF=0.05;

B13=-EQI0^2*KPD/M;          B23=-EQI0*KqD/TDI0;        B42=DVREF*KA/TA;
B43=-KA*EQI0*KVD/TA;        B52=DVREF*KA*KF/(TA*TF);
B53=-KA*KF*EQI0*KVD/(TA*TF); B71=DWREF*KIN; B73=KWD;

MATRIX   B1

C1=[1 0 0 0 0 0 0 0];  C2=[0 KV0 0 0 0 0 0 0]; C3=[0 2*KP0*EQI0 0 0 0 0 0 0];
C4=[0 2*KQ0*EQI0 0 0 0 0 0 0]; C5=[0 0 0 0 0 0 0 1];  C6=[0 1 0 0 0 0 0 0];
```

```
TMF=6;
TM1=[1:0.01:TMF]; D1=[0 0 0]; D2=[0 0 EQI0*KVD]; D3=[0 0 EQI0^2*KPD];
D4=[0 0 EQI0^2*KQD]; D5=[0 0 0]; D6=[0 0 0];

PLOT RESULTS
------------------------------------------------------------
Q1=[10 0 0 0 0 0 0 0;
  0 1 0 0 0 0 0 0;
  0 0 1 0 0 0 0 0;
  0 0 0 1 0 0 0 0;
  0 0 0 0 1 0 0 0;
  0 0 0 0 0 1 0 0;
  0 0 0 0 0 0 1 0;
  0 0 0 0 0 0 0 1];

R1=[1  0  0; 0  0.2 0; 0  0  0.2];

[KK1,S,E]=lqr(A1,B1,Q1,R1); % KK1 is the state feedback matrix of G-1 (size=3x8)
AK1=A1-B1*KK1;

PLOT RESULTS
```

% GENERATOR # 2 SIMULATION

```
VD0=P0*VT0/sqrt(P0^2+(Q0+VT0^2/XQ)^2);
IQ0=VD0/XQ; VQ0=sqrt(VT0^2-VD0^2);
ID0=(P0-IQ0*VQ0)/VD0;
EQI0=VQ0+XDI*ID0;
DEL0=asin(VD0/VT0);
EFD0=EQI0+(XD-XDI)*ID0;
PM0=P0;
RL0=VT0^2*P0/(P0^2+Q0^2); XL0=VT0^2*Q0/(P0^2+Q0^2);
KI0=1/(RL0^2+(XQ+XL0)*(XDI+XL0));
KP0=KI0^2*RL0*XQ*(XL0+XQ)+KI0*RL0-KI0^2*RL0*XDI*(XL0+XQ);
KQ0=KI0*(XL0+XQ)-KI0^2*XDI*(XL0+XQ)^2-KI0^2*RL0^2*XQ;
KV0=sqrt(KI0^2*RL0^2*XQ^2+(1-KI0*XDI*(XL0+XQ))^2);
Kq0=1+KI0*(XD-XDI)*(XL0+XQ);

P=0.8; Q=0.2334;
RL=VT0^2*P/(P^2+Q^2);       XL=VT0^2*Q/(P^2+Q^2);
KI=1/(RL^2+(XQ+XL)*(XDI+XL));
KP=KI^2*RL*XQ*(XL+XQ)+KI*RL-KI^2*RL*XDI*(XL+XQ);
KQ=KI*(XL+XQ)-KI^2*XDI*(XL+XQ)^2-KI^2*RL^2*XQ;
KV=sqrt(KI^2*RL^2*XQ^2+(1-KI*XDI*(XL+XQ))^2);
Kq=1+KI*(XD-XDI)*(XL+XQ);
KPD=KP-KP0; KQD=KQ-KQ0; KVD=KV-KV0; KqD=Kq-Kq0;

m1=KG*DMP/M-KIN-DMP^2*KD/(M^2);
m2=(2*KG*KP0*EQI0-2*KD*DMP*KP0*EQI0)/M+(2*KD*KP0*EFD0-4*KD*KP0*Kq0*EQI0)/(M*TDI0);
m3=2*KD*KP0*EQI0/(M*TDI0);
m4=(KG*EQI0^2-DMP*KD*EQI0^2)/M+(2*KD*EQI0*EFD0-2*KD*Kq0*EQI0^2)/(M*TDI0);
m5=2*KD*EQI0^2*KP0/(M*TDI0);
m6=K1*KD/(M*T2)+2*K1*KD/(M*T1);
m7=K1*K2*K3*KD/(M*T2);
m8=KD*DMP/(M^2)-KG/M+2*KD/(M*T1);
KWD=m4*KPD-m5*KqD;

A11=-DMP/M;                 A12=-2*KP0*EQI0/M;         A18=1/M;
A22=-Kq0/TDI0;              A23=1/TDI0;                A33=-(KE+SE)/TE;
A34=1/TE;                   A42=-KA*KV0/TA;            A44=-1/TA;
A45=-KA/TA;                 A52=-KA*KV0*KF/(TA*TF);    A54=-KF/(TA*TF);
A55=-(KA*KF+TA)/(TA*TF);    A66=-1/T2;                 A67=K2*K3/T2;
```

```
A71=m1;                 A72=m2;                 A73=m3;
A76=-m6;                A77=m7;                 A78=m8;
A86=K1/T2+2*K1/T1;      A87=-K1*K2*K3/T2;       A88=-2/T1;

MATRIX   A2

DWREF=0.05; DVREF=0.05;

B13=-EQI0^2*KPD/M;   B23=-EQI0*KqD/TDI0;
B42=DVREF*KA/TA;     B43=-KA*EQI0*KVD/TA;
B52=DVREF*KA*KF/(TA*TF); B53=-KA*KF*EQI0*KVD/(TA*TF);
B71=DWREF*KIN; B73=KWD;

MATRIX   B2

C1=[1 0 0 0 0 0 0 0]; C2=[0 KV0 0 0 0 0 0 0]; C3=[0 2*KP0*EQI0 0 0 0 0 0 0];
C4=[0 2*KQ0*EQI0 0 0 0 0 0 0]; C5=[0 0 0 0 0 0 0 1]; C6=[0 1 0 0 0 0 0 0];
D1=[0 0 0]; D2=[0 0 EQI0*KVD]; D3=[0 0 EQI0^2*KPD];
D4=[0 0 EQI0^2*KQD]; D5=[0 0 0]; D6=[0 0 0];

D1=[0 0 0]; D2=[0 0 0]; D3=[0 0 0]; D4=[0 0 0]; D5=[0 0 0]; D6=[0 0 0];

PLOT RESULTS
----------------------------
Q2=[  10 0  0  0  0  0  0  0:
   0  1  0  0  0  0  0  0;
   0  0  1  0  0  0  0  0;
   0  0  0  1  0  0  0  0;
   0  0  0  0  1  0  0  0;
   0  0  0  0  0  1  0  0;
   0  0  0  0  0  0  1  0;
   0  0  0  0  0  0  0  1];

D1=[0 0 0];   D2=[0 0 0];   D3=[0 0 0];   D4=[0 0 0];   D5=[0 0 0];   D6=[0 0 0];
R2=[1  0  0; 0  0.2 0; 0  0  0.2];
R2X=[1  0; 0  0.2];

[KK2X,S,E]=lqr(A2,B2X,Q2,R2X); %  KK2X is the state feedback matrix of G-2 (size=2x8)
AK2X=A2-B2X*KK2X;
[KK2,S,E]=lqr(A2,B2,Q2,R2);
AK2=A2-B2*KK2

PLOT RESULTS

%  G1 & G2 DE-CENTRALIZED CONTROL:
K01=zeros(size(KK1)); K02=zeros(size(KK2X));
  KDC=[KK1  K01;
      K02  KK2X]; % De-centralized control state feedback matrix "KDC"
% Note !!: This KDC matrix will be used later in this program !!

% TWO PARALLEL OPERATING DIESEL GENERATORS 1 & 2
% DATA FOR THE TWO PARALLEL GENERATORS:
  VT0=1; DMP=0;
%  Power factor = 0.8 as base
%  Total initial load is P=1000 KW, Q=550 KVAR,
%  P1B=1100KW (1375KVA), P2B=1360KW(1700KVA)
%  Initially the 1000KW=0.91 p.u is shared equally as follows:
P01=0.3252;  P02=0.40207; Q0=0.4;
P0=P01+P02;
VD0=P0*VT0/sqrt(P0^2+(Q0+(1/XQ1+1/XQ2)*VT0^2)^2);
VQ0=sqrt(VT0^2-VD0^2);
IQ01=VD0/XQ1;                    IQ02=VD0/XQ2;
```

```
ID01=(P01-VQ0*IQ01)/VD0;
Q01=VQ0*ID01-VD0*IQ01;          Q02=Q0-Q01;
ID02=(P02-VQ0*IQ02)/VD0;
Q0=Q01+Q02;
EQI01=VQ0+XDI1*ID01;            EQI02=VQ0+XDI2*ID02;
EFD01=EQI01+(XD1-XDI1)*ID01;     EFD02=EQI02+(XD2-XDI2)*ID02;
VR01=(KE1+SE1)*EFD01;           VR02=(KE2+SE2)*EFD02;
VREF1=EFD01/KA1+VT0;            VREF2=EFD02/KA2+VT0;
RLi=VT0^2*P0/(P0^2+Q0^2);        XLi =VT0^2*Q0/(P0^2+Q0^2);
X1i =XQ1+XLi+XLi*XQ1/XQ2;        X2i =RLi+RLi*XDI1/XDI2;
X5i =XDI1+XLi+XLi*XDI1/XDI2;     X6i =RLi+RLi*XQ1/XQ2;
X7i =X1i/(X1i*X5i+X2i*X6i);      X8i =X6i/(X1i*X5i+X2i*X6i);
X9i =X7i+X7i*XLi/XDI2+X8i*RLi/XDI2; X10i=X7i*XLi/XDI2+X8i*RLi/XDI2;
X11i=(1/X6i)*(1+XLi/XDI2-X5i*X9i); X12i=(1/X6i)*(X5i*X10i-XLi/XDI2);
X13i=XQ1*X11i/XQ2;               X14i=XQ1*X12i/XQ2;
X15i=(1/XDI2)*(XDI1*X9i-1);      X16i=(1/XDI2)*(1-XDI1*X10i);
X17i=XQ1*X11i;                   X18i=XQ1*X12i;
X19i=1-XDI1*X9i;                 X20i=XDI1*X10i;
X21i=X17i*X9i+X19i*X11i;
X22i=X18i*X9i-X17i*X10i+X19i*X12i+X20i*X11i;
X23i=X20i*X12i-X18i*X10i;        X24i=X17i*X15i+X19i*X13i;
X25i=X17i*X16i+X18i*X15i+X19i*X14i+X20i*X13i;
X26i=X18i*X16i+X20i*X14i;
X27i=XDI1*X9i-XD1*X9i-1;         X28i=XD1*X10i-XDI1*X10i;
X29i=XDI2*X15i-XD2*X15i;         X30i=XDI2*X16i-XD2*X16i-1;
X31i=2*X21i*X27i/TDI01+X22i*X29i/TDI02;
X32i=(2*X21i*X28i+X22i*X27i)/TDI01+(X22i*X30i+2*X23i*X29i)/TDI02;
X33i=X22i*X28i/TDI01+2*X23i*X30i/TDI02; X34i=2*X21i/TDI01;
X35i=X22i/TDI01;                 X36i=X22i/TDI02;
X37i=2*X23i/TDI02;               X38i=2*X24i*X27i/TDI01+X25i*X29i/TDI02;
X39i=(2*X24i*X28i+X25i*X27i)/TDI01+(X25i*X30i+2*X26i*X29i)/TDI02;
X40i=X25i*X28i/TDI01+2*X26i*X30i/TDI02;
X41i=2*X24i/TDI01;               X42i=X25i/TDI01;
X43i=X25i/TDI02;                 X44i=2*X26i/TDI02;
X45i=X9i*X19i-X11i*X17i;         X46i=X9i*X20i-X10i*X19i-X11i*X18i-X12i*X17i;
X47i=X10i*X20i+X12i*X18i;        X48i=X15i*X19i-X13i*X17i;
X49i=X16i*X19i+X15i*X20i-X14i*X17i-X13i*X18i; X50i=X16i*X20i-X14i*X18i;

P1=1.4545; Q1=Q0; % Total Load is increased from 1000 KW to 2750 KW;
RL=VT0^2*P1/(P1^2+Q1^2);         XL=VT0^2*Q1/(P1^2+Q1^2);
X1 =XQ1+XL+XL*XQ1/XQ2;           X2 =RL+RL*XDI1/XDI2;
X5 =XDI1+XL+XL*XDI1/XDI2;        X6 =RL+RL*XQ1/XQ2;
X7 =X1/(X1*X5+X2*X6);            X8 =X6/(X1*X5+X2*X6);
X9 =X7+X7*XL/XDI2+X8*RL/XDI2;    X10=X7*XL/XDI2+X8*RL/XDI2;
X11=(1/X6)*(1+XL/XDI2-X5*X9);    X12=(1/X6)*(X5*X10-XL/XDI2);
X13=XQ1*X11/XQ2;                 X14=XQ1*X12/XQ2;
X15=(1/XDI2)*(XDI1*X9-1);        X16=(1/XDI2)*(1-XDI1*X10);
X17=XQ1*X11;                     X18=XQ1*X12;
X19=1-XDI1*X9;                   X20=XDI1*X10;
X21=X17*X9+X19*X11;
X22=X18*X9-X17*X10+X19*X12+X20*X11;
X23=X20*X12-X18*X10;             X24=X17*X15+X19*X13;
X25=X17*X16+X18*X15+X19*X14+X20*X13;
X26=X18*X16+X20*X14;
X27=XDI1*X9-XD1*X9-1;            X28=XD1*X10-XDI1*X10;
X29=XDI2*X15-XD2*X15;            X30=XDI2*X16-XD2*X16-1;
X31=2*X21*X27/TDI01+X22*X29/TDI02;
X32=(2*X21*X28+X22*X27)/TDI01+(X22*X30+2*X23*X29)/TDI02;
X33=X22*X28/TDI01+2*X23*X30/TDI02;  X34=2*X21/TDI01;
X35=X22/TDI01;                   X36=X22/TDI02;
X37=2*X23/TDI02;                 X38=2*X24*X27/TDI01+X25*X29/TDI02;
X39=(2*X24*X28+X25*X27)/TDI01+(X25*X30+2*X26*X29)/TDI02;
```

```
X40=X25*X28/TDI01+2*X26*X30/TDI02;
X41=2*X24/TDI01;                X42=X25/TDI01;
X43=X25/TDI02;                  X44=2*X26/TDI02;
X45=X9*X19-X11*X17;             X46=X9*X20-X10*X19-X11*X18-X12*X17;
X47=X10*X20+X12*X18;            X48=X15*X19-X13*X17;
X49=X16*X19+X15*X20-X14*X17-X13*X18;  X50=X16*X20-X14*X18;

 KP11=2*X21i*EQI01+X22i*EQI02;          KP12=X22i*EQI01+2*X23i*EQI02;
 KP21=2*EQI01*X24i+X25i*EQI02;          KP22=X25i*EQI01+2*EQI02*X26i;
 KP1D=EQI01^2*(X21-X21i)+EQI01*EQI02*(X22-X22i)+EQI02^2*(X23-X23i);
 KP2D=EQI01^2*(X24-X24i)+EQI01*EQI02*(X25-X25i)+EQI02^2*(X26-X26i);
 Kq1D=EQI01*(X27-X27i)+EQI02*(X28-X28i); Kq2D=EQI01*(X29-X29i)+EQI02*(X30-X30i);
 KQ11=2*X45i*EQI01+X46i*EQI02;          KQ12=X46i*EQI01-2*EQI02*X47i;
 KQ21=2*EQI01*X48i+X49i*EQI02;          KQ22=X49i*EQI01+2*EQI02*X50i;
 KQ1D=EQI01^2*(X45-X45i)+EQI01*EQI02*(X46-X46i)-EQI02^2*(X47-X47i);
 KQ2D=EQI01^2*(X48-X48i)+EQI01*EQI02*(X49-X49i)+EQI02^2*(X50-X50i);
 KV1=VD0*X17i/VT0+VQ0*X19i/VT0;         KV2=VD0*X18i/VT0+VQ0*X20i/VT0;
 KVD=VD0*EQI01*(X17-X17i)/VT0+VD0*EQI02*(X18-X18i)/VT0+VQ0*EQI01*(X19-
X19i)/VT0+VQ0*EQI02*(X20-X20i)/VT0;

m11 = KG1*DMP/M1-KIN1-KD1*DMP^2/M1^2;
m21 = KG1*KP11/M1-KD1*DMP*KP11/M1^2+KD1*KP11*X27i/(M1*TDI01)+KD1*KP12*X29i/(M1*TDI02);
m31 = KP12*KG1/M1-KD1*DMP*KP12/M1^2+KD1*KP11*X28i/(M1*TDI01)+KD1*KP12*X30i/(M1*TDI02);
m41 = KD1*KP11/(M1*TDI01);
m51 = KD1*KP12/(M1*TDI02);
m61 = (KD1/M1)*(K11/T21+2*K11/T11);
m71 = K11*KD1*K21*K31/(M1*T21);
m81 = 2*KD1/(M1*T21)+KD1*DMP/M1^2-KG1/M1;
KW1D= (KD1/M1)*(-DMP*KP1D/M1+KP11*Kq1D/TDI01+KP12*Kq2D/TDI02)+KG1*KP1D/M1;

m12 = KG2*DMP/M2-KIN2-KD2*DMP^2/M2^2;
m22 = KG2*KP21/M2-KD2*DMP*KP21/M2^2+KD2*KP21*X27i/(M2*TDI01)+KD2*KP22*X29i/(M2*TDI02);
m32 = KP22*KG2/M2-KD2*DMP*KP22/M2^2+KD2*KP21*X28i/(M2*TDI01)+KD2*KP22*X30i/(M2*TDI02);
m42 = KD2*KP21/(M2*TDI01);
m52 = KD2*KP22/(M2*TDI02);
m62 = (KD2/M2)*(K12/T22+2*K12/T12);
m72 = K12*KD2*K22*K32/(M2*T22);
m82 = 2*KD2/(M2*T22)+KD2*DMP/M2^2-KG2/M2;
KW2D= KG2*KP2D/M2+KD2*KP21*Kq1D/(M2*TDI01)+KD2*KP22*Kq2D/(M2*TDI02)-KD2*KP2D*DMP/M2^2;

A11=-DMP/M1; A12=-KP11/M1; A18=1/M1; A110=-KP12/M1;
A22=X27i/TDI01; A23=1/TDI01; A210=X28i/TDI01;
A33=-(KE1+SE1)/TE1; A34=1/TE1;
A42=-KA1*KV1/TA1; A44=-1/TA1; A45=-KA1/TA1; A410=-KA1*KV2/TA1;
A52=-KA1*KF1*KV1/(TA1*TF1); A54=-KF1/(TA1*TF1); A55=-(KA1*KF1+TA1)/(TA1*TF1);
A510=-KA1*KF1*KV2/(TA1*TF1);
A66=-1/T21; A67=K21*K31/T21;
A71=m11; A72=m21; A73=m41; A76=-m61; A77=m71; A78=m81; A710=m31; A711=m51;
A86=K11/T21+2*K11/T11; A87=-K11*K21*K31/T21; A88=-2/T11;
A92=-KP21/M2; A99=-DMP/M2; A910=-KP22/M2; A916=1/M2;
A102=X29i/TDI02; A1010=X30i/TDI02; A1011=1/TDI02;
A1111=-(KE2+SE2)/TE2; A1112=1/TE2;
A122=-KA2*KV1/TA2; A1210=-KA2*KV2/TA2; A1212=-1/TA2; A1213=-KA2/TA2;
A132=-KA2*KF2*KV1/(TA2*TF2); A1310=-KA2*KF2*KV2/(TA2*TF2); A1312=-KF2/(TA2*TF2);
A1313=-(KA2*KF2+TA2)/(TA2*TF2);
A1414=-1/T22; A1415=K22*K32/T22;
A159=m12; A152=m22; A153=m42; A1510=m32; A1511=m52; A1514=-m62; A1515=m72; A1516=m82;
A1614=K12/T22+2*K12/T12; A1615=-K12*K22*K32/T22; A1616=-2/T12;

MATRIX  AP

B15=-KP1D/M1;  B95=-KP2D/M2;          B25=Kq1D/TDI01;
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```
B105=Kq2D/TDI02;
B42=0.05*KA1/TA1;          B45=-KA1*KVD/TA1;          B124=0.05*KA2/TA2;
B125=-KA2*KVD/TA2;
B52=0.05*KA1*KF1/(TA1*TF1); B55=-KA1*KF1*KVD/(TA1*TF1); B134=0.05*KA2*KF2/(TA2*TF2);
B135=-KA2*KF2*KVD/(TA2*TF2);
B71=0.05*KIN1;          B75=KW1D;          B153=0.05*KIN2;
B155=KW2D;

MATRIX  BP

C1 =[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];          % G-1 freq
C2 =[0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0];          % G-2 freq
C3 =[0 KV1 0 0 0 0 0 0 0 KV2 0 0 0 0 0 0 0];       % Terminal voltage
C4 =[0 KP11 0 0 0 0 0 0 0 KP12 0 0 0 0 0 0 0];     % G-1 Pe
C5 =[0 KP21 0 0 0 0 0 0 0 KP22 0 0 0 0 0 0 0];     % G-2 Pe
C7 =[0 KQ11 0 0 0 0 0 0 0 KQ12 0 0 0 0 0 0 0]; % G-1 Qe
C8 =[0 KQ21 0 0 0 0 0 0 0 KQ22 0 0 0 0 0 0 0]; % G-2 Qe
CP=[C1;C2;C3;C4;C5;C7;C8];
D0=[0 0 0 0 0]; D3=[0 0 0 0 KVD];          D4=[0 0 0 0 KP1D];     D5=[0 0 0 0 KP2D];
D7=[0 0 0 0 KQ1D];  D8=[0 0 0 0 KQ2D];
C=[C1;C2;C3;C4;C5;C7;C8];  D=[D0;D0;D3;D4;D5;D7;D8];

QP=[10 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0;
  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0;
  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0;
  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0;
  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0;
  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0;
  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0;
  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0;
  0  0  0  0  0  0  0  0  10 0  0  0  0  0  0  0  0;
  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0;
  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0;
  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0;
  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0;
  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0;
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0;
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0;
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1];

RP=[1  0  0  0  0; 0  0.2 0  0  0; 0  0  1  0  0; 0  0  0  0.2 0; 0  0  0  0  0.2];

[KP,S,E]=lqr(AP,BP,QP,RP);

KCC  = KP;      % Centralized Control State Feedback Matrix " KCC "  ;
AKDC = AP-BP*KDC; % De-Centralized Control State Matrix " AKDC "     ;
AKCC = AP-BP*KCC; % Centralized Control State Matrix " AKCC "        ;

PLOT RESULTS
```

LQR STABILIZER DESIGN FOR G-1 & G-2 TO REPLACE BOTH CENTRALIZED AND DE-CENTRALIZED CONTROLLERS
The following algorithm is referenced to "Multi-machine power system stabilizer design based on new LQR approach IEE Proc-Gen Transm-Dist, Vol.142,No.5,Sept 1995

'Q' Matrix determination:

```
WC        = gram(AP,BP); WO=gram(AP',CP');
[Ab,Bb,Cb,G,T] = BALREAL(AP,BP,CP);
WCb       = gram(Ab,Bb);
WOb       = gram(Ab',Cb');
SEGMA     = diag(WCb);
QbV       = [1 SEGMA(1)/SEGMA(2) SEGMA(1)/SEGMA(3) SEGMA(1)/SEGMA(4) SEGMA(1)/SEGMA(5)
             SEGMA(1)/SEGMA(6) SEGMA(1)/SEGMA(7) SEGMA(1)/SEGMA(8)  SEGMA(1)/SEGMA(9)
```

```
                SEGMA(1)/SEGMA(10)  SEGMA(1)/SEGMA(11) SEGMA(1)/SEGMA(12) SEGMA(1)/SEGMA(13)
                SEGMA(1)/SEGMA(14) SEGMA(1)/SEGMA(15) 0];
   Qb          = diag(QbV);
   Q           = T'*Qb*T;
```

% 'R' Matrix determination:

```
   IN          = eye(16);
   BP1         = [0;0;0;0;0;0;B71;0;0;0;0;0;0;0;0;0];
   BP2         = [0;0;0;B42;B52;0;0;0;0;0;0;0;0;0;0;0];
   BP3         = [0;0;0;0;0;0;0;0;0;0;0;0;0;0;B153;0];
   BP4         = [0;0;0;0;0;0;0;0;0;0;0;B124;B134;0;0;0];
   BP5         = [B15;B25;0;B45;B55;0;B75;0;B95;B105;0;B125;B135;0;B155;0];
```

%>>>> **PURTURBATION OF INPUTS**:

```
   BP1B  = BP1+0.015*rand(16,1);
   BP2B  = BP2+0.015*rand(16,1);
   BP3B  = BP3+0.015*rand(16,1);
   BP4B  = BP4+0.015*rand(16,1);
   BP5B  = BP5+0.015*rand(16,1);
```

%>>>> **TRANSFERRING THE SYSTEM INTO ORDERED BALANCED FORM**:

```
  [Ab1,Bb1,Cb1] = BALREAL(AP,BP1B,IN);
  [Ab2,Bb2,Cb2] = BALREAL(AP,BP2B,IN);
  [Ab3,Bb3,Cb3] = BALREAL(AP,BP3B,IN);
  [Ab4,Bb4,Cb4] = BALREAL(AP,BP4B,IN);
  [Ab5,Bb5,Cb5] = BALREAL(AP,BP5B,IN);
```

%>>>> **COTROLLABILITY & OBSERVABILITY GRAMIANS**:

```
WCb1 = gram(Ab1,Bb1);  WOb1 = gram(Ab1',Cb1);
  WCb2 = gram(Ab2,Bb2);  WOb2 = gram(Ab2',Cb2);
  WCb3 = gram(Ab3,Bb3);  WOb3 = gram(Ab3',Cb3);
  WCb4 = gram(Ab4,Bb4);  WOb4 = gram(Ab4',Cb4);
  WCb5 = gram(Ab5,Bb5);  WOb5 = gram(Ab5',Cb5);
```

%>>>> **CONTRIBUTION OF THE 5 INPUTS**:

```
  W1 = sum(diag(WCb1));
  W2 = sum(diag(WCb2));
  W3 = sum(diag(WCb3));
  W4 = sum(diag(WCb4));
  W5 = sum(diag(WCb5));
```

%>>>> **TIGHTNESS OF THE CONTROL ACTION**:

```
ITA=W1/SEGMA(1);
ITA=0;
```

%>>>> **'R' MATRIX**:

```
  V=[1 W2/W1 W3/W1 W4/W1 W5/W1];
  R=ITA*diag(V);
%------------------------------------------------------------------------
% LQR STABILIZER USING PARALLEL GENERATOR MODEL MATRIX 'AP'
% AND CALCULATED Q & R MATRICES:
%-----------------------------
  [KPQR,S,E]=lqr(AP,BP,Q,R);
% Another form of centralized control !!
  KCC  = KPQR;
  APCC = AP-BP*KCC;  PLOT RESULTS
```

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